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A multi-objective MILP model for the design and operation of future integrated multi-vector energy networks capturing detailed spatio-temporal dependencies

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HIGHLIGHTS

- MILP model for strategic design & tactical operation of multi-vector energy networks.
- Evolution of integrated natural gas, electricity, hydrogen and syngas networks to 2050.
- Multi-objective: min cost, max profit, min emission, max renewable energy production.
- Optimal combination of conversion & storage technologies & transport infrastructures.
- When & where to invest in facilities; what resources to use, how to transport & store.

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ABSTRACT

A multi-objective optimisation model, based on mixed integer linear programming, is presented that can simultaneously determine the design and operation of any integrated multi-vector energy networks. It can answer variants of the following questions:

> What is the most effective way, in terms of cost, value/profit and/or emissions, of designing and operating the integrated multi-vector energy networks that utilise a variety of primary energy sources to deliver different energy services, such as heat, electricity and mobility, given the availability of primary resources and the levels of demands and their distribution across space and time? When to invest in technologies, where to locate them; what resources should be used, where, when and how to convert them to the energy services required; how to transport the resources and manage inventory?

Scenarios for Great Britain were examined involving different primary energy sources, such as natural gas, biomass and wind power, in order to satisfy demands for heat, electricity and mobility via various energy vectors such as electricity, natural gas, hydrogen and syngas. Different objectives were considered, such as minimising cost, maximising profit, minimising emissions and maximising renewable energy production, subject to the availability of suitable land for biomass and wind turbines as well as the maximum local production and import rates for natural gas.

Results suggest that if significant mobility demands are met by hydrogen-powered fuel cell vehicles, then hydrogen is the preferred energy vector, over natural gas, for satisfying heat demands. If natural gas is not used and energy can only be generated from wind power and biomass, electricity and syngas are the preferred energy carriers for satisfying electricity and heat demands.

1. Introduction

Traditionally, energy networks evolved independently, with fossil fuels as the dominant primary resource and electricity and natural gas as the energy vectors. As we strive to move towards a more sustainable and low-carbon future energy system, a much greater variety of primary sources of energy (such as wind, solar and biomass) and more technologies with different types and scales for generating,

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transmitting, distributing and storing energy will be utilised. A change in the energy mix can be expected and other energy vectors, such as hydrogen, syngas, methanol etc., may also play important roles. Each of these energy vectors is capable of delivering multiple energy service demands, such as heat, electricity and mobility. Similarly, an energy service demand can be satisfied through different energy vectors. For example, hydrogen, electricity, natural gas and biofuels are all potential alternatives to petroleum for meeting mobility demands. Furthermore, with the higher penetration of renewables, all of the networks need to be aware of the intermittent supply from renewables, covering for shortfalls and allowing full utilisation when supply exceeds demand. Integrating the networks for different energy vectors can improve the efficiency of the whole energy system and also increase the penetration of renewables.

There are many complex issues associated with the integration of energy networks and mathematical modelling is a valuable tool to help understand them. Mathematical models (hereafter called "models") can provide an accurate representation of the potential technologies, infrastructures and resources that may become part of the network. Through computational experiments, the behaviour of the system can be explored at the national level over a long future planning horizon. Using optimisation techniques, the best design among the many possible alternatives can be determined – this involves selecting the appropriate combinations of technologies for resource conversion, storage, transmission and distribution, when to invest in them, where to locate them and what their capacities should be. Models can aid in determining the most effective way of operating the system and formulating control strategies to ensure that the operation is robust in the presence of disturbances and uncertainties. They can also provide a holistic understanding of the system, which could help inform policy on the future shape of the energy sector as a whole.

Motivated by the desire to develop efficient and sustainable systems that can deliver the energy needs of today's and future societies, the aim is to develop a mathematical model that can simultaneously determine the best design and operation of the integrated multi-vector energy networks to obtain the most value from limited available resources. One of the main challenges is that primary energy resources are available at different quantities, at different times and at different locations. The demands for energy services are also distributed in space and time but often not matched to the availability of primary resources. Therefore, the model needs to be sufficiently detailed to account for the distribution of resources across space and time, the interactions between different networks and energy vectors and the operational issues at different time scales (e.g. accounting for hourly variation, differences between days of the week, seasonality and long-term planning and investment).

There are a number of different modelling approaches to planning energy systems at national scale that employ mathematical programming but most of them are not suitable for optimising the design and operation of integrated multi-vector energy networks. These models typically fall into two very broad categories: equilibrium models, such as MARKAL/TIMES [1,2] and all of its variants, and energy supply chain models (also known as network models) based on a multi-echelon supply chain representation. An extensive review of these models appears in our previous publications [3–5] and is summarised in the following paragraphs.

Equilibrium models are typically steady state, multi-period models that consider how economics, supply and demand change over a number of planning periods, e.g. years. Although they can represent a large number of conversion technologies, their major weakness is that they are not spatially-resolved, have little or no temporal detail below the planning periods and do not contain a detailed (or typically any) representation of the energy transmission/distribution networks or of energy storage. Various temporal MARKAL type models [6] feature "time slicing", reflecting different time periods with different demand and renewable supply patterns. However, dynamics cannot be considered because these periods are not linked and therefore operational issues such as storage and ramp-up/ramp-down rates of technologies cannot be modelled; storage is only considered by shifting some demands to a *user-selected* time interval and assuming sufficient storage capacity to support this. Therefore this family of models is not suited to solving the complex problem of designing integrated multi-vector energy networks, in which one must consider in detail: the transmission and distribution of energy (hence a high spatial resolution is required); the detailed operation of the network, which requires a fine temporal resolution to account for operational issues; and new interactions between the networks when they become more distributed and include a higher penetration of intermittent supply technologies, as can be expected to occur. Storage may also be expected to play a key role in supporting these highly integrated networks with intermittent supply of energy; this also requires a fine temporal resolution and a model that can predict the dynamics of the system and track the inventory of stored energy over time so that the storage facilities can be sized correctly.

Energy supply chain models, on the other hand, typically include nodes and edges to represent the spatial dependence of the system. Nodes represent locations of entities in the chain (e.g. production sites, conversion technologies, storage facilities) and edges represent transport connections between the nodes. Although there are many energy supply chain models in the literature, almost all of these models are based on manufacturing supply chains so they have a multi-echelon structure that breaks down the supply chain into a number of stages or echelons (e.g. for hydrogen networks, typical echelons include primary resources/raw materials, production plants, storage facilities and distribution centres, which are very similar to the echelons of manufacturing supply chains). In this representation, the direction of the flow of resources across the echelons is specified or fixed before the optimisation (e.g. from primary resources to production plants to storage facilities to distribution), which means that the resources can only flow in the specified direction. For example, in the model presented by Almansoori and Shah [7–9] for hydrogen supply chains, on which many energy supply chain models are based (e.g. [10–22]), the resources from the production plants will always have to go to the storage facilities and cannot be transported to other regions or distributed directly to the customers. Also, the reverse pathways cannot be handled by the multi-echelon formulation and adding a technology (e.g. fuel cell, which will define the reverse pathway of converting hydrogen back to electricity), will require a significant change to the *core mathematical structure* of the model. This inflexibility makes the multi-echelon formulation unsuitable for modelling integrated multi-vector energy networks. A suitable model will need to be able to decide at any given time what to do with a particular resource in order to optimise the whole system: converting it to another resource vs. holding it in a storage facility vs. transmitting it to another location vs. distributing it customers to satisfy demands. At the same time, the model needs to determine what form of energy is most suitable for transportation and storage. A further limitation of existing supply chain models is their representation of time: while many of these models can consider a long-term horizon, they are multi-period (e.g. each period represents the average over a five-year period); all of them lack the shorter time scale to capture the seasonality of energy service demands and availability of renewable sources and even finer time scale to account for the intermittency of renewable sources and dynamics of energy storage, which requires at least an hour-by-hour account of the operation of the network and an inventory balance for storage.

There are also other recent studies that considered multi-vector energy networks, but with only gas and electricity as energy carriers; other carriers such as hydrogen and syngas are not part of the system. For example, Chaudry and co-workers developed an NLP model to optimise the operation of integrated gas and electricity networks of a fixed design [23], which was later extended to include capacity expansion [24]. Devlin et al. [25] presented an MILP model for unit commitment and dispatch of gas and electricity networks, also only considering operation for a fixed network design. Martínez Cesena et al. [26] presented an MILP model for operation of a multi-vector microgrid (over one day at half-hourly intervals for 7 characteristic day types) and performed a Monte Carlo simulation to determine the reliability of the system. The same group also considered low-carbon technologies in multi-vector systems [27]. Liu and Mancarella [28] proposed a simulation model of an integrated heat, power and gas network for district energy systems and applied it to a case study for the University of Manchester campus.

The main contribution of this work is the novel and powerful mixed integer linear programming (MILP) model that can simultaneously determine the design and operation of *any* integrated multi-vector energy networks comprising technologies for conversion, storage and transport. The model takes into account the spatial distribution and temporal variability of system properties (such as demands and availability of resources) and determines the spatial structure of the integrated networks (e.g. what technologies and infrastructures to invest in at each planning period, where and what capacity) and their hourly operation over the entire planning horizon. It can consider the most general situation of resources being converted to any other, including recycles (i.e. circular chains), and resources can be stored and transported at any stage in the chain. It can also capture the intermittency of renewables and dynamics of energy storage: it has a detailed model of storage including an inventory balance and is thus able to optimise the inventory of each resource in each storage facility at every hour over the entire planning horizon. Storage technologies are characterised using separate properties for charging, maintaining and discharging inventory rather than round-trip properties used in other models. In addition, land footprint constraints are applied to technologies and the harvesting/production of available natural resources, based on a detailed GIS analysis using technical and socio-environmental constraints. The model is formulated as a multi-objective optimisation problem, therefore it can be used to explore the trade-offs between different conflicting objectives (e.g. minimisation of cost, maximisation of profit, minimisation of CO_2 emissions, maximisation of energy production or different combinations of these). Furthermore, the model is data-driven so any resources, technologies and infrastructures can be included in the network superstructure (which defines all of the potential pathways from primary resources to energy services) by simply including them as elements in the sets and providing the required data for their parameters. In particular, the model is not restricted to two or three vectors as other multi-vector models are: diverse vectors and primary resources with very different properties and time scales can be considered. To the authors' knowledge, no other model presented in the literature combines all of these necessary features.

The structure of this paper is as follows: the problem is defined in Section 2 and the key model features required to solve the problem are discussed in Section 3. The network superstructure is described in Section 4 and the complete MILP formulation is presented in Section 5. The applicability of the model is illustrated through a number of case studies, discussed in Section 6, considering Great Britain (GB) as the region of interest. Finally, the conclusions and insights are summarised in Section 7.

2. Problem statement

The model can solve problems of the following type:

Given:

- Spatio-temporal demands for resources and energy services
- Spatio-temporal availability of primary energy sources and raw materials
- Characteristics of each technology (e.g. CAPEX, O&M, efficiency, lifetime)

Determine:

- Network design
  - Location, number and capacity of generation/conversion and storage technologies
  - Structure of transport infrastructure network (transmission and distribution)
  - When and where to purchase/install the technologies
  - What interactions there are between the networks/energy vectors
- Network operation
  - Which resources to convert, store and transport (how much, where and when)
  - Which technologies to use at different times
  - Transport flows between different regions

Subject to:

- Demand satisfaction
- Conservation and other physical laws
- Constraints on resources (e.g. land, water), costs and GHG emissions
- Technological constraints (e.g. availability of technologies, build rates)
- Social and political constraints (e.g. siting of specific technologies)

Objective:

- Minimise cost
- Minimise environmental impact (e.g. GHG emissions)
- Maximise value (e.g. profit)
- Maximise energy production
- Any combination of the above

3. Key elements of the model

In order to solve the problem outlined in Section 2, the model requires five key components: time, space, resources, technologies and infrastructures.

3.1. Time

The time resolution of the model needs to capture simultaneously the long-term strategic decisions (e.g. investments in and retirements of technologies and infrastructures), as well as short-term operational issues (e.g. intermittency and dynamics of energy storage). The main challenge in using contiguous hourly intervals over a long planning horizon (e.g. more than one year) is that it results in a very large model that could take a very long time to solve or even be intractable [3]. Therefore, instead of using contiguous hourly intervals, the time domain is divided into different subdomains of varying granularity: hourly intervals to capture the intermittency of renewable energy sources and dynamics of energy storage; day types to take into account that, for example, the demands for energy services on the weekdays could be different from those on the weekends; seasons to consider the seasonal variations in energy service demands and availability of primary energy sources; and yearly/decadal periods to model strategic investment decisions.

In Fig. 1, h, d, t and y are the elements in the sets of hourly intervals $\mathbb{H}$, day types $\mathbb{D}$, seasons $\mathbb{T}$ and yearly intervals $\mathbb{Y}$, respectively. As shown in the figure, the intervals in each time level need not be uniform. In fact, a more computationally efficient representation (i.e. one with fewer decision variables and constraints) can be obtained by using non-uniform time intervals and exploiting the periodicity in system properties. For example, if each hourly interval h has a duration of n_h^{hd} hours, such that $\Sigma_h n_h^{\mathrm{hd}} = 24$, then a daily demand profile can be represented more efficiently by using more and shorter duration intervals during peak times to capture the dynamics and fewer and longer duration intervals when the demands are relatively static. Similarly, a

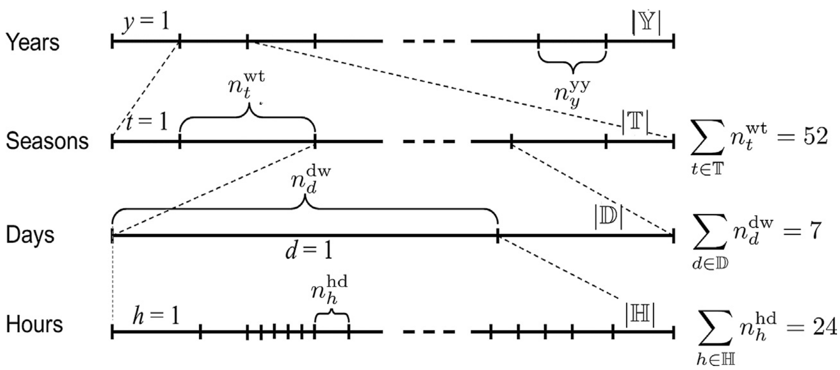


Fig. 1. Temporal representation that efficiently captures long-term strategic decisions (e.g. investment in technologies and infrastructures) and short-term operational issues (e.g. intermittency of renewables, dynamics of energy storage).

weekly demand profile can be represented by a sequence of demand profiles for each day type d, each repeated by n_d^{dw} times, such that $\Sigma_d n_d^{\text{dw}} = 7$. For example, a week's worth of data can be represented by 5 repetitions of a typical weekday profile followed by 2 repetitions of a typical weekend profile. In the same way, each season t is composed of n_t^{wt} identical weeks, such that $\Sigma_t n_t^{\text{wt}} = 52$, and each yearly interval y comprises n_y^{yy} identical years.

As the storage inventory over time needs to be contiguous, additional constraints are required to link the storage inventory within and between the different time levels. Furthermore, because the model is dynamic, additional variables are needed for the initial inventories (i.e. similar to the required initial conditions when solving differential equations). Section 5.4 describes how these are formulated in the model.

3.2. Space

The region of study is divided into a number of zones and each zone, $z \in \mathbb{Z}$, may:

- be of any shape and size;
- have dynamic demands for various resources;
- contain primary resources that are available in varying quantities over time;
- host technologies for conversion and storage of resources;
- be connected with other cells via transport infrastructures;
- import or export resources.

The appropriate spatial and temporal resolutions of the model depend on the problem being considered and there is often a trade-off between them. For example, when modelling bioenergy networks, a high spatial resolution is needed because biomass yields are strongly dependent on location. In addition, regional transport of biomass by trucks is effective at a distance of no longer than 100 km. Therefore, a 50 km × 50 km grid, as shown in Fig. 2(a), which is the spatial resolution used in the Biomass Value Chain Model [4], strikes a good compromise between capturing these important spatial effects and obtaining a solution within reasonable time. In this problem, the important temporal effects are the seasonal variations in crop production and energy service demands, as well as the yearly (or decadal) investments in technologies and infrastructures, and change in climate. Therefore, the model should include seasonal and yearly intervals. A resolution at the hourly level may provide little benefit and may result in an intractable model because of the high spatial resolution.

In other cases, such as networks utilising intermittent renewables, such as wind energy, it is necessary to use a very high temporal resolution in order to capture the intermittency of renewables and dynamics of energy storage. In order for the model to capture these effects and remain tractable, a more aggregated spatial representation is necessary. Fig. 2(b), which is based on the National Grid's study zones [29], is often useful when modelling energy networks in GB because there are many available datasets aggregated at this level (e.g. from the Net Electricity Trading Agreement website [30]).

The spatial resolution shown in Fig. 2(b) is used in the case studies presented in Section 6.

3.3. Resources

Each resource, $r \in \mathbb{R}$, can be converted to a different resource, transported to another zone, stored or used to satisfy energy service demands. Resources may represent any of the following:

- primary energy sources, such as natural gas, biomass, wind and sunlight;
- intermediates, which are produced by a technology and then can be consumed by a different technology; e.g. syngas and pyrolysis oil;
- end-use vectors, which are used to satisfy energy service demands: e.g. electricity, heat, transport fuels;
- by-products, which are valuable resources that are produced with the end-use vectors; and
- wastes, such as low-grade heat and GHG emissions.

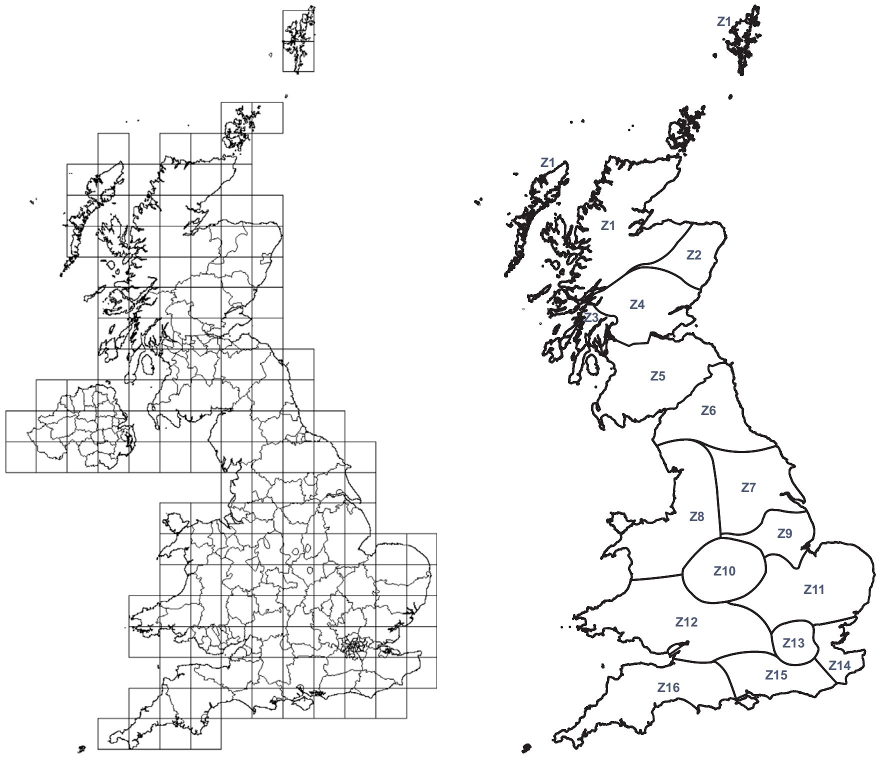
3.4. Technologies

The relationship between different resources and technologies can be represented using a Value Web Diagram (VWD), which is a powerful method of representing *any* technology.

In general, there are three types of technology in any network: conversion, storage and transport technologies. The VWD representation for each technology type is illustrated in Fig. 3, where resources are represented by the circles, conversion technologies are indicated by the rectangles, storage technologies are denoted by the pentagons, transport technologies are represented by the hexagons and the possible pathways are indicated by the arrows.

Conversion technologies transform one set of resources into another set of resources. In the simple example shown in Fig. 3(a), technologies Tech1 and Tech2 are interacting: the outputs from Tech1 are inputs to Tech2 and vice versa. Here, the conversion pathways form a *circular* chain. This is one of the strengths of this representation: by including the technologies that can interconvert resources in both directions, the model can determine the optimal form of a resource (energy or material) at any given time and location. For example, if both electrolysers and fuel cells are part of the network superstructure, the model can determine when and where it is best to have the energy in the form of electricity or hydrogen.

The VWD representation for storage technologies is presented in Fig. 3(b), where the three stages for storing resource r_1 are modelled as *tasks*: "put", "hold" and "get". In the diagram, the "put" task transfers r_1 from the zone to the store, requiring some r_2 and producing some wastes r_3 (e.g. CO_2). The "hold" task maintains r_1 in storage, which could be at less than 100% efficiency, the losses being converted to r_3; this task may also require some r_2. Finally, the "get" task retrieves r_1 from storage and



(a) (b)

Fig. 2. Example spatial representations: a high spatial resolution, such as (a) the 50 km square grid, is recommended when spatial effects are more important to capture than temporal effects; and an aggregated spatial representation, such as (b) based on the National Grid's study zones [29], is recommended when the intermittency of renewables and dynamics of energy storage are to be modelled.

delivers it to the zone, requiring some r_4. This representation allows different costs, efficiencies, losses and resource requirements to be applied to each stage of storage.

Fig. 3(c) shows the VWD representation for transport technologies. In the diagram, resource r_2 is transported from zone z to zone z', requiring resource r_1 from zone z (e.g. fuel for road transport or compression power for pipeline transport) and generating waste r_3 (e.g. CO_2 emissions) in both cells. This representation enables losses, resource requirements, emissions and costs associated with transporting a resource to be explicitly considered in the model.

3.5. Infrastructures

In the model, transport technologies are considered distinctly from transport infrastructures: the former represent the processes by which resources are moved from one cell to another cell, while the latter represent physical structures that support the transport technologies. For example, truck (or train) transport of biomass pellets is the transport technology and road (or railway) is the infrastructure; flow of fluid (or flow of electricity) is the transport process or "technology" and pipeline (or electricity cable) is the infrastructure. The reason for modelling the transport technologies distinctly from the infrastructures is that some technologies may share the same infrastructure. For example, road tankers for petrol, LNG, hydrogen all use the same road infrastructure, therefore, if a new infrastructure is built then all transport processes that use the road are enabled. Establishing a separate road infrastructure for each truck transport process would overestimate the cost, not to mention be unrealistic. Note that in most cases the existing road/rail infrastructure will be included in the scenario and there will be no need to build new road/rail connections. This functionality is useful when considering greenfield (urban) developments.

4. Network superstructure

The basic VWDs for conversion, storage and transport technologies, illustrated in Fig. 3, can be combined in order to model more realistic problems, such as the one depicted in Figs. 4 and 5, where different primary resources, such as natural gas, biomass and wind power, can be converted to different energy vectors such as syngas, hydrogen and electricity, in order to satisfy different energy service demands, such as heat, electricity and mobility.

In Fig. 4, the circles represent the resources and the rectangles represent the conversion technologies, which convert sets of input resources (indicated by the arrows directed into the rectangles) to sets of output resources (arrows directed out). The grey circles indicate resources with demands and the pentagons represent storage technologies (simplified in this diagram but the model naturally includes the put, hold and get tasks, illustrated in Fig. 3(b), for each storage device).

The primary energy resources are biomass, shown at the top of the diagram, natural gas, in the middle, and electricity, to the far right. Biomass can be grown in any of the zones, subject to constraints on available area in each zone. Natural gas, assumed to be at a pressure of 7 MPa, is available in certain zones and can also be imported into zones that contain natural gas terminals. Finally electricity can be produced from wind turbines, based on the number of turbines installed in each zone (subject to available area, calculated using the site suitability criteria) and the wind speed. Two pressure states are considered for gaseous resources: one at 7 MPa, which is the maximum pipeline pressure, and another at 20 MPa, which is the storage pressure.

Biomass can be converted to syngas at 7 MPa by the gasification technology. The syngas can then be converted to electricity, heat or both via the four technology families to the right: CCGT, CHP, boiler and domestic boiler. There are two *distinct* CCGT technologies: one that consumes syngas and one that consumes natural gas. To avoid cluttering the diagram, the two rectangles are shown stacked. Similarly for the syngas, natural gas and hydrogen CHP technologies; there are also three distinct technologies for boilers and domestic boilers that are syngas-, natural gas- or hydrogen-fired.

The CCGT technologies produce electricity from either syngas or natural gas. The CHP technologies produce both electricity and district

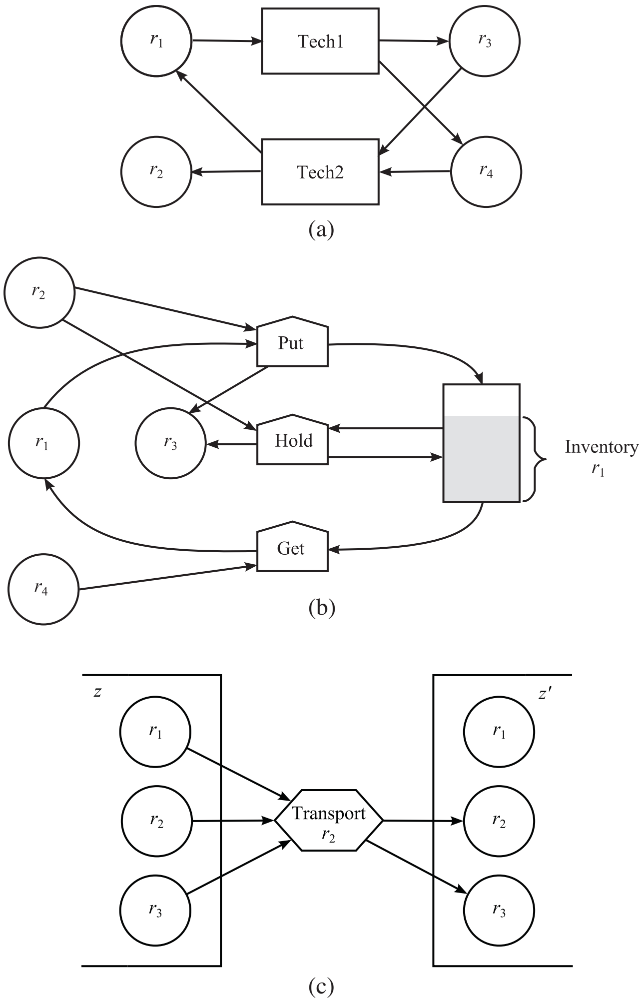


Fig. 3. Value Web representations for (a) conversion technologies, (b) storage technologies and (c) transport technologies. These are the basic building blocks that can be combined to form a value web, representing the integrated network superstructure.

heat. The boiler technologies (at commercial/industrial scale) produce district heat and the domestic boilers produce heat. The district heat produced by the large-scale CHP and boilers can be converted to heat to satisfy domestic heat demands via a district heating network technology.

Finally heat demands can also be satisfied by domestic electric heaters.

Hydrogen at 7 MPa can be produced from natural gas by the SMR technology. This hydrogen can then be converted to electricity in a fuel cell (in addition to the CHP and boiler routes to electricity and/or heat) or it can be distributed to fuelling stations to satisfy demands for hydrogen from fuel cell vehicles (these demands are indicated by the grey circle labelled "Mobility").

To the left of the diagram are the storage technologies, which are assumed to store syngas, natural gas or hydrogen at 20 MPa. To convert between the two pressure levels, there are dedicated compressors and expanders. Compressors also require electricity to operate, which is indicated by a minus sign (indicating consumption of electricity) and a circled E in the bottom right-hand corner of the rectangles. Expanders can recover some of the potential energy of the compressed gas as electricity, indicated by the plus sign and circled E. The circled E also appears in the electricity resource to indicate that the circled E refers to that particular resource. This compact symbology avoids the use of many additional arrows that would adversely affect the legibility of the diagram.

One final route is that from electricity to hydrogen via electrolysers. The electrolyser technology, at the bottom of the diagram, is assumed to produce hydrogen at 20 MPa. The input water and output oxygen resources are not considered here but can easily be included if desired.

Note that since the expanders and compressors are explicitly considered as conversion technologies, the put and get tasks of the corresponding storage technologies will be simple input-output relations and will not include electricity as a resource requirement.

Three different sizes for each technology were considered in order to capture economies of scale. These were not shown in Figs. 4 and 5 to avoid cluttering the diagram.

The gaseous resources at 7 MPa can be transported between zones via pipelines (see Fig. 5) and electricity can be transported via a number of different transmission lines (single-circuit and double-circuit HVAC and HVDC overhead lines and underground cables).

The network superstructure can be easily expanded by simply adding more resources and technologies and the required data for their parameters without the need to change the mathematical formulation, which is discussed in the next section.

5. Mathematical model

Indices and sets

$b \in \mathbb{B}$	Transport infrastructures
$c \in \mathbb{C} \subset \mathbb{R}$	Biomass resources ("crops")
$d \in \mathbb{D}$	Day types
$\mathbb{E} \subseteq \mathbb{R}$	End vectors
$f \in \mathbb{F}$	Flow directions
$i \in \mathbb{I}$	Performance metrics (e.g. costs, CO_2 emissions)
$h \in \mathbb{H}$	Hourly intervals
$l \in \mathbb{L}$	Transport technologies
$p \in \mathbb{P}$	Conversion technologies
$\mathbb{P}^{D} \subseteq \mathbb{P}$	Domestic conversion technologies
$\mathbb{P}^{C} \subseteq \mathbb{P}$	Commercial/industrial conversion technologies
$r \in \mathbb{R}$	Resources
$s \in \mathbb{S}$	Storage facilities
$t \in \mathbb{T}$	Seasons
$y \in \mathbb{Y}$	Five-year planning periods
$\tilde{y} \in \tilde{\mathbb{Y}}$	Contiguous yearly intervals (used in discounting costs)
$z \in \mathbb{Z}$	Transmission zones

Parameters

$A_{zy}^{W,max}$	Maximum suitable land area for wind turbines in zone z in planning period y [m^2]
$A_{zy}^{Bio,max}$	Maximum available land area for biomass in zone z in planning period y [ha]
a_{sz}	Parameter to restrict the location of storage facilities, such as indicating the presence of underground storage in zone z. $a_{sz} = 1$ if storage facility s can be established in zone z, 0 otherwise.
BR_{py}	Maximum total number of conversion technologies p that can be built in planning period y (build rate)
b_b^{max}	Maximum capacity of infrastructure b [MW]
C_{biy}^{B}	Unit capital impact of infrastructure b in planning period y [£/(connection-km) or $kgCO_2$/(connection-km)]
C_{piy}^{P}	Unit capital impact of conversion technology p in planning period y [£ or $kgCO_2$]
C_{siy}^{S}	Unit capital impact of storage facility s in planning period y [£ or $kgCO_2$]
C_{iy}^{W}	Unit capital impact of wind turbine in planning period y [£ or $kgCO_2$]
c_{city}^{Bio}	Unit impact of producing biomass c in season t of planning period y [£/MWh or $kgCO_2$/MWh] (planting, cultivation and harvesting impacts)

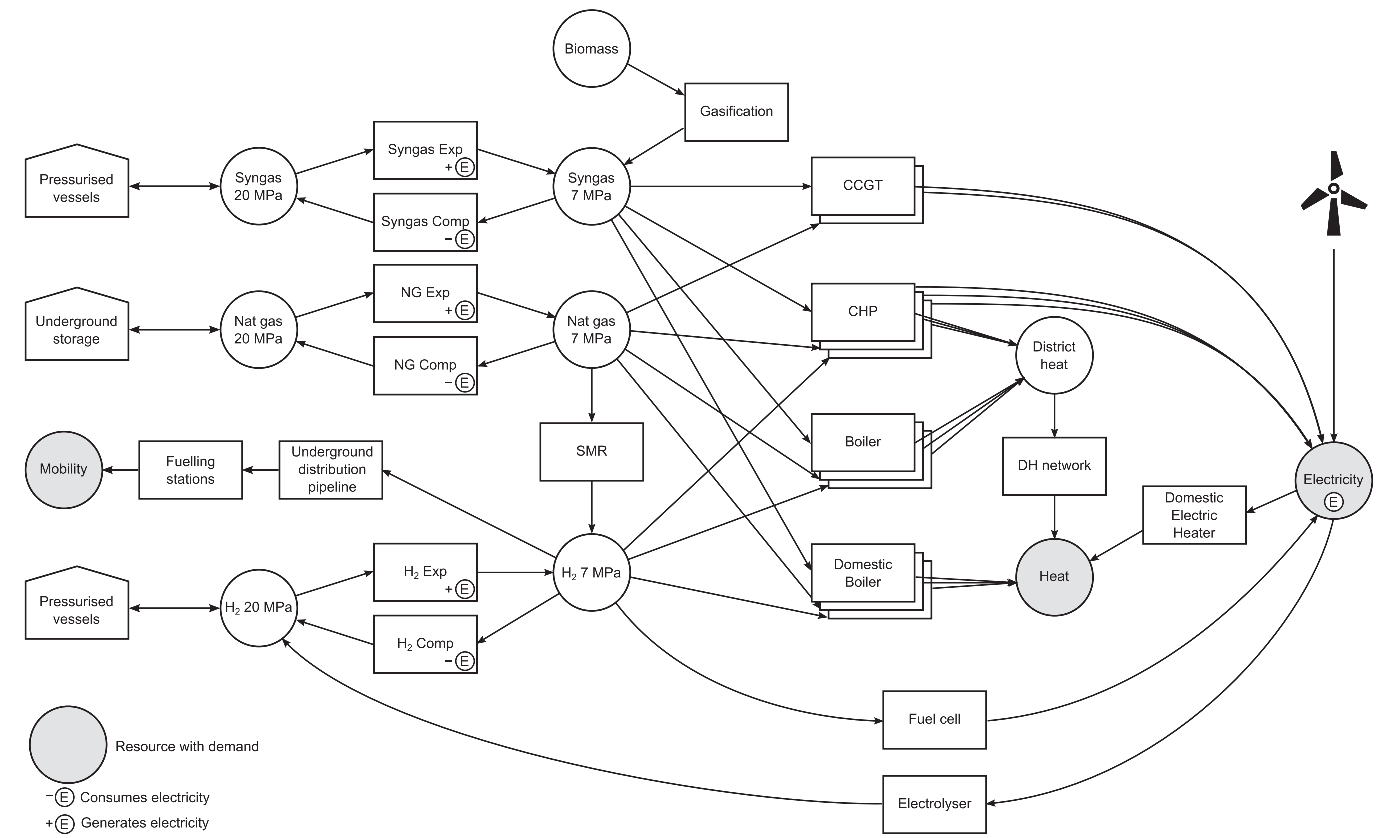


Fig. 4. Superstructure of the integrated multi-vector energy networks.

c^{M}_{rihdty} Unit impact of importing resource r during hour h, day type d, season t and planning period y [£/MWh or $kgCO_2$/MWh]

c^{U}_{rihdty} Unit impact of producing resource r during hour h, day type d, season t and planning period y (e.g. local natural gas production) [£/MWh or $kgCO_2$/MWh]

c^{X}_{rihdty} Unit impact of exporting resource r during hour h, day type d, season t and planning period y [£/MWh or $kgCO_2$/MWh]

$D^{\mathrm{C}}_{\star iy}$ Factor that discounts the capital impact in planning period y back to 2015 ($\star \in \{b,p,s\}$ for transport infrastructures, conversion technologies and storage technologies, respectively)

D^{OM}_{iy} Factor that discounts the O&M impact in planning period y back to 2015

D^{W}_{iy} Factor that discounts the capital impact of new wind turbines invested in planning period y back to 2015

$D^{\mathrm{act}}_{rzhdty}$ Actual demand for resource r in zone z during hour h, day type d, season t and planning period y [MW]

$D^{\mathrm{comp}}_{rzhdty}$ Compulsory demand for resource r in zone z during hour h, day type d, season t and planning period y [MW] (must always be satisfied)

$D^{\mathrm{opt}}_{rzhdty}$ Optional demand for resource r in zone z during hour h, day type d, season t and yearly period y [MW] (can be satisfied if economically/environmentally beneficial or when maximising energy production)

$d_{zz'}$ Distance between the demand centres of zones z and z' [km]

f^{loc}_{zy} Fraction of suitable area for biomass in zone z that can be used in planning period y

f^{nat}_{y} Fraction of total (national) suitable area for biomass that can be used in planning period y

LB_{lb} = 1 if transport technology l can use infrastructure b, 0 otherwise

$m^{\max}_{rzhdty}$ Maximum rate of import of resource r in zone z during hour h, day type d, season t and planning period y [MW]

N^{EW}_{zy} Number of existing on-shore wind turbines in zone z in planning period y (accounts for estimated retirement dates)

n^{hd}_{h} Duration of hourly interval h [h]

n^{dw}_{d} Number of times day type d occurs in a week

n^{wt}_{t} Number of repeated weeks in season t

n^{yy}_{y} Number of repeated years in planning period y

N^{EPC}_{pz} Number of existing commercial conversion technologies of type p in zone z

NR^{EPC}_{pzy} Number of existing commercial conversion technologies of type p in zone z that retire at the beginning of planning period y

N^{ES}_{sz} Number of existing storage technologies of type s in zone z

NR^{ES}_{szy} Number of existing storage technologies of type s in zone z that retire at the beginning of planning period y

$N^{\mathrm{EB}}_{bzz'}$ Number of existing transport infrastructure connections of type b between zones z and z'

$p^{\max}_{p}$ Maximum production rate of technology p [MW]

$p^{\min}_{p}$ Minimum production rate of technology p [MW]

$q^{\max}_{l}$ Max transfer/flow rate of each transport technology l [MW]

R^{EW} Radius of existing on-shore wind turbines [m]

R^{W} Radius of new on-shore wind turbines [m]

$RT^{\mathrm{P}}_{py'y}$ = 1 if conversion technology p invested in at the beginning of planning period y' retires at the beginning of planning period y, 0 otherwise

$RT^{\mathrm{S}}_{sy'y}$ = 1 if storage facility s invested in at the beginning of planning period y' retires at the beginning of planning period y, 0 otherwise

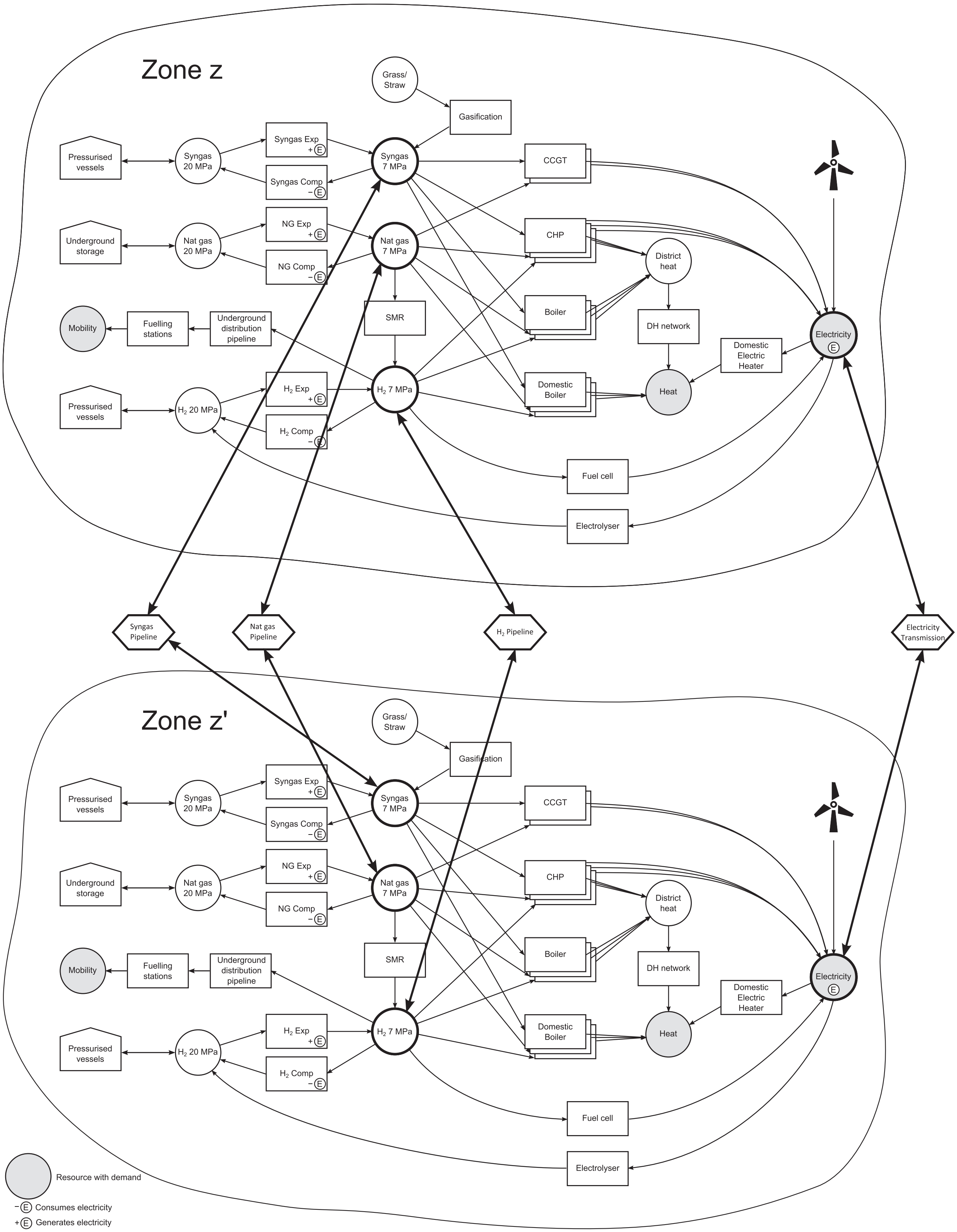


Fig. 5. Transmission technologies (in bold outline) connect the networks between different zones.

$RT^{W}_{y'y}$ = 1 if wind turbines invested in at the beginning of planning period y' retire at the beginning of planning period y, 0 otherwise
$s_s^{get,max}$ Maximum rate of withdrawal from storage facility s [MW]
$s_s^{hold,max}$ Maximum storage capacity of a single storage facility s [MWh]
$s_s^{put,max}$ Maximum rate of injection into storage facility s [MW]
V_{riy} Unit value (e.g. price) of resource r in planning period y
v_{zhdty} Wind speed in zone z during hour h of day type d in season t of planning period y [m/s]
x_z x-coordinate of the centre of demand of zone z
y_z y-coordinate of the centre of demand of zone z
Y^{Bio}_{czty} Yield potential for biomass c in zone z for season t of planning period y [MWh/ha/season]
α_{rpy} Conversion factor of resource r in technology p in planning period y
β_b Indicates bidirectionality of transport infrastructure b: $=-1$ if unidirectional i.e. A to B only; = 0 if independent bidirectional, i.e. A to B and B to A need separate links; = 1 if bidirectional
ϵ Weight for total energy production in objective function
γ Finance rate
η Efficiency of the wind turbines
ι Discount rate
$\lambda_\star$ Economic lifetime of technologies [year] ($\star \in \{b,p,s\}$ for transport infrastructures, conversion technologies and storage technologies, respectively)
$\nu_{zz'}$ Binary parameter equal to 1 if zone z is adjacent to zone z'
ρ^{air} Air density [kg/m^3]
σ^{get}_{srfy} Conversion factor when withdrawing resource r from storage facility s in planning period y
σ^{hold}_{srfy} Conversion factor when holding resource r in storage facility s in planning period y
σ^{put}_{srfy} Conversion factor when putting resource r into storage facility s in planning period y
ς Factor that converts cost from £ to £M and CO_2 emissions from kg to Mkg (to ensure the model is well-scaled)
$\bar{\tau}_{lrfy}$ Distance-independent conversion factor for transport technology l transporting resource r in planning period y
$\hat{\tau}_{lrfy}$ Distance-dependent conversion factor for transport technology l transporting resource r in planning period y
ϕ^{B}_{biy} Annual O&M impact of transport infrastructure b in planning period y [(£ or kgCO_2)/(connection-km-yr)]
ϕ^{P}_{piy} Annual O&M (fixed) impact of conversion technology p in planning period y [£/yr or kgCO_2/yr]
ϕ^{S}_{siy} Annual O&M (fixed) impact of storage facility s in planning period y [£/yr or kgCO_2/yr]
ϕ^{W}_{iy} Annual O&M (fixed) impact of wind turbines in planning period y [£/yr or kgCO_2/yr]
φ^{P}_{piy} Unit variable operating impact of conversion technology p in planning period y [£/MWh or kgCO_2/MWh]
$\hat{\varphi}^{Q}_{liy}$ Distance-dependent unit variable operating impact of transport process l in planning period y [£/km/MWh or kgCO_2/km/MWh]
$\bar{\varphi}^{Q}_{liy}$ Distance-independent unit variable operating impact of transport process l in planning period y [£/MWh or kgCO_2/MWh] (e.g. flat rate freight charges)
φ^{SG}_{siy} Unit variable operating impact for withdrawing inventory from storage facility s in planning period y [£/MWh or kgCO_2/MWh]
φ^{SH}_{siy} Unit variable operating impact for holding inventory in storage facility s in planning period y [£/MWh or kgCO_2/MWh]
φ^{SP}_{siy} Unit variable operating impact for putting inventory into storage facility s in planning period y [£/MWh or kgCO_2/MWh]
χ^{max}_{rzhdty} Maximum rate of export of resource r from zone z during hour h, day type d, season t and planning period y [MW]
ω_i Weight for key performance indicator i in objective function

Positive variables

A^{Bio}_{czy} Area allocated to production of biomass (crop) c in zone z during planning period y [ha]
D^{sat}_{rzhdty} Optional demands satisfied in zone z during hour h of day type d in season t of planning period y [MW]
I_{szhdty} Inventory in storage facility s in zone z during hour h of day type d in season t of planning period y [MWh]
$I^{0,act}_{szdty}$ Inventory in storage facility s in zone z at the start of day type d of season t in planning period y [MWh]
$I^{0,sim}_{szdty}$ Inventory in storage facility s in zone z at the start of the simulated cycle for day type d of season t in planning period y [MWh]
$\mathscr{I}^{P}_{iy}$ Total net present impact of building new conversion technologies in planning period y [£M or MkgCO_2]
$\mathscr{I}^{S}_{iy}$ Total net present impact of building new storage technologies in planning period y [£M or MkgCO_2]
$\mathscr{I}^{Q}_{iy}$ Total net present impact of building new transport infrastructures in planning period y [£M or MkgCO_2]
$\mathscr{I}^{W}_{iy}$ Total net present capital impact of building new wind turbines in planning period y [£M or MkgCO_2]
$\mathscr{I}^{Bio}_{iy}$ Total net present impact of producing biomass in planning period y [£M or MkgCO_2]
$\mathscr{I}^{m}_{iy}$ Total net present impact of importing resources in planning period y [£M or MkgCO_2]
$\mathscr{I}^{fp}_{iy}$ Total net present fixed O&M impact of conversion technologies in planning period y [£M or MkgCO_2]
$\mathscr{I}^{fq}_{iy}$ Total net present fixed O&M impact of transport infrastructures in planning period y [£M or MkgCO_2]
$\mathscr{I}^{fs}_{iy}$ Total net present fixed O&M impact of storage technologies in planning period y [£M or MkgCO_2]
$\mathscr{I}^{Rev}_{iy}$ Total net present revenue from the sales of energy services for satisfying demands in planning period y [£M or MkgCO_2]
$\mathscr{I}^{U}_{iy}$ Total impact of utilising natural resources in planning period y [£M or MkgCO_2]
$\mathscr{I}^{vp}_{iy}$ Total net present variable operating impact of production facilities in planning period y [£M or MkgCO_2]
$\mathscr{I}^{vs}_{iy}$ Total net present variable operating impact of storage facilities in planning period y [£M or MkgCO_2]
$\mathscr{I}^{vq}_{iy}$ Total net present variable operating impact of transport technologies in planning period y [£M or MkgCO_2]
$\mathscr{I}^{w}_{iy}$ Total net present O&M impact of wind turbines in planning period y [£M or MkgCO_2]
$\mathscr{I}^{x}_{iy}$ Total net present impact of exporting resources in planning period y [£M or MkgCO_2]
M_{rzhdty} Rate of import of resource r in zone z during hour h of day type d in season t of planning period y [MW]
N^{PD}_{pzy} Millions of domestic conversion technology $p \in \mathbb{P}^{D}$ in zone z at the beginning of (and throughout) planning period y
U_{rzhdty} Utilisation of resource r in zone z during hour h of day type d in season t of planning period y [MW]
u^{max}_{rzhdty} Maximum availability of resource r in zone z during hour h of day type d in season t of planning period y [MW]
X_{rzhdty} Rate of export of resource r in zone z during hour h of day type d in season t of planning period y [MW]

$\mathscr{P}_{pzhdty}$ Total utilisation rate of conversion technology p in zone z during hour h of day type d in season t of planning period y [MW]

$\mathscr{Q}_{lzz'hdty}$ Rate of operation of transport technology l from zone z to zone z' during hour h of day type d in season t of planning period y [MW]

$\mathscr{S}^{\text{get}}_{szhdty}$ Rate at which inventory is withdrawn from storage s in zone z during hour h of day type d in season t of planning period y [MW]

$\mathscr{S}^{\text{hold}}_{szhdty}$ Rate at which inventory is held in storage s in zone z during hour h of day type d in season t of planning period y [MW]

$\mathscr{S}^{\text{put}}_{szhdty}$ Rate at which inventory is added to storage s in zone z during hour h of day type d in season t of planning period y [MW]

Free variables

P_{rzhdty} Net rate of production of resource r in zone z during hour h of day type d in season t of planning period y [MW]

Q_{rzhdty} Net rate of transport of resource r into zone z from all other zones during hour h of day type d in season t of planning period y [MW]

S_{rzhdty} Net rate storage of resource r in zone z during hour h of day type d in season t of planning period y [MW]

Z Objective function

$\delta^{\text{d}}_{szdty}$ Net accumulation of inventory s in zone z in day type d in season t of planning period y [MWh]

δ^{t}_{szty} Net accumulation of inventory s in zone z in season t of planning period y [MWh]

δ^{y}_{szy} Net accumulation of inventory s in zone z in planning period y [MWh]

Integer variables

$N^{\text{B}}_{bzz'y}$ Number of transport infrastructure b built between zones z and z' at the beginning of (and available for use throughout) planning period y

N^{PC}_{pzy} Total number of commercial conversion technology $p \in \mathbb{P}^{\text{C}}$ in zone z at the beginning of (and throughout) planning period y

N^{S}_{szy} Total number of storage technology s in zone z at the beginning of (and throughout) planning period y

N^{W}_{zy} Total number of new wind turbines in zone z at the beginning of (and throughout) planning period y

NU^{EW}_{zy} Total number of existing wind turbines being used in zone z during planning period y

$NI^{\text{B}}_{bzz'y}$ Number of additional transport infrastructure b invested in at the beginning of planning period y between zones z and z'

NI^{PC}_{pzy} Number of additional commercial conversion technology $p \in \mathbb{P}^{\text{C}}$ invested in at the beginning of planning period y in zone z

NI^{S}_{szy} Number of additional storage facility s invested in at the beginning of planning period y in zone z

NI^{W}_{zy} Number of additional wind turbines invested in at the beginning of planning period y in zone z

NR^{PC}_{pzy} Number of commercial conversion technology $p \in \mathbb{P}^{\text{C}}$ retired in zone z at the beginning of planning period y

NR^{S}_{szy} Number of storage facility s retired in zone z at the beginning of planning period y

NR^{W}_{zy} Number of wind turbines retired in zone z at the beginning of planning period y

5.1. *Resource balance, maximum availability and utilisation*

The resource balance is:

$$U_{rzhdty} + M_{rzhdty} + P_{rzhdty} + S_{rzhdty} + Q_{rzhdty} \geqslant D^{\text{comp}}_{rzhdty} + D^{\text{sat}}_{rzhdty} + X_{rzhdty} \quad (1)$$

$$\forall\, r \in \mathbb{R}, z \in \mathbb{Z}, h \in \mathbb{H}, d \in \mathbb{D}, t \in \mathbb{T}, y \in \mathbb{Y}$$

where U_{rzhdty} is the rate of utilisation of resource r in zone z during hour h day type d season t and planning period y, M_{rzhdty} is the rate of import of resource r, P_{rzhdty} is the net rate of production of resource r due to the operation of the conversion technologies, S_{rzhdty} is the net rate of utilisation of resource r from the storage technologies in zone z, Q_{rzhdty} is the net rate of transport of resource r into zone z from all other zones, D^{comp}_{rzhdty} and D^{sat}_{rzhdty} are the demands for resource r and X_{rzhdty} is the rate of export of resource r. The inequality in the constraint helps with the feasibility: since there is a minimum part-load constraint on the conversion technologies, there can be some cases where energy production exceeds demands and capacity for storage, and the inequality allows overproduction to be curtailed.

Demands for resources are split into two categories: those that must always be satisfied, D^{comp}_{rzhdty}, and those that may optionally be satisfied, D^{opt}_{rzhdty}. This approach is useful when considering part of the whole energy system (such as wind power only, renewables only or biomass only) and the objective of the optimisation is to determine how much energy can be produced from this supply sector and which demands are best satisfied. The model can therefore determine how much of each of these demands is satisfied:

$$D^{\text{sat}}_{rzhdty} \leqslant D^{\text{opt}}_{rzhdty} \quad \forall\, r \in \mathbb{R}, z \in \mathbb{Z}, h \in \mathbb{H}, d \in \mathbb{D}, t \in \mathbb{T}, y \in \mathbb{Y} \quad (2)$$

The rate of utilisation of resource r cannot exceed the resource availability:

$$U_{rzhdty} \leqslant u^{\max}_{rzhdty} \quad \forall\, r \in \mathbb{R}-\mathbb{C}, z \in \mathbb{Z}, h \in \mathbb{H}, d \in \mathbb{D}, t \in \mathbb{T}, y \in \mathbb{Y} \quad (3)$$

The maximum wind power availability is a function of the number and size of wind turbines and the hourly wind speed:

$$u^{\max}_{\text{Elec},zhdt} = 0.5 \times 10^{-6} \eta \rho^{\text{air}} [N^{\text{W}}_{zy} \pi (R^{\text{W}})^2 + NU^{\text{EW}}_{zy} \pi (R^{\text{EW}})^2] \tilde{v}^3$$

$$\forall\, z \in \mathbb{Z}, h \in \mathbb{H}, d \in \mathbb{D}, t \in \mathbb{T}, y \in \mathbb{Y} \quad (4)$$

where $\tilde{v}_{zhdty}$ is the effective wind speed defined by:

$$\tilde{v}_{zhdty} = \begin{cases} v_{zhdty} & \text{if } v_{\text{cut-in}} \leqslant v_{zhdty} \leqslant v_{\text{cut-out}} \\ 0 & \text{otherwise.} \end{cases} \quad (5)$$

The number of existing wind turbines utilised in the network cannot exceed the number of existing wind turbines (i.e. the model can choose whether to use existing wind turbines or not):

$$NU^{\text{EW}}_{zy} \leqslant N^{\text{EW}}_{zy} \quad \forall\, z \in \mathbb{Z}, y \in \mathbb{Y} \quad (6)$$

The number of new wind turbines that can be installed is governed by the land footprint constraint (Eq. (7)), which ensures that the land area occupied by the new wind turbines does not exceed the maximum available suitable area (assuming a spacing of five rotor diameters between wind turbines):

$$\pi (5R^{\text{W}})^2 N^{\text{W}}_{zy} \leqslant A^{\text{W,max}}_{zy} \quad \forall\, z \in \mathbb{Z}, y \in \mathbb{Y} \quad (7)$$

The maximum suitable land area for wind turbines, $A^{\text{W,max}}_{zy}$, was determined by overlaying different technical and environmental constraints using GIS software in order to identify the suitable sites or parcels of land. The land area already occupied by existing wind turbines was excluded from $A^{\text{W,max}}_{zy}$.

For natural gas, the hourly availability, $u^{\max}_{\text{NG},zhdty}$, was obtained from the National Grid's gas transmission operational data [31].

The availability of biomass on the other hand is on a seasonal level and is equal to the allocated area multiplied by the yield potential. The total amount of biomass utilised in each season cannot exceed this seasonable availability:

$$\sum_{hd} U_{czhdty} n^{\text{hd}}_h n^{\text{dw}}_d n^{\text{wt}}_t \leqslant A^{\text{Bio}}_{czy} Y^{\text{Bio}}_{czty} \quad \forall\, c \in \mathbb{C}, z \in \mathbb{Z}, t \in \mathbb{T}, y \in \mathbb{Y} \quad (8)$$

For biomass, the land footprint constraints can be specified at the local and/or national level: Eq. (9) limits the allocated area for biomass production to the specified maximum fraction of suitable area in each zone while Eq. (10) limits it to the maximum fraction of the total suitable area at the national level.

$$\sum_{c} A^{\mathrm{Bio}}_{czy} \leqslant f^{\mathrm{loc}}_{zy} A^{\mathrm{Bio,max}}_{zy} \quad \forall\, z \in \mathbb{Z}, y \in \mathbb{Y} \tag{9}$$

$$\sum_{cz} A^{\mathrm{Bio}}_{czy} \leqslant f^{\mathrm{nat}}_{y} \sum_{z} A^{\mathrm{Bio,max}}_{zy} \quad \forall\, y \in \mathbb{Y} \tag{10}$$

The maximum suitable land area for biomass, $A^{\mathrm{Bio,max}}_{zy}$, was determined by performing land suitability analysis using GIS software.

The land suitability criteria for biomass and wind turbines are discussed in Section 6 where different scenarios for Great Britain are explored in detail.

5.2. Staged technology investments and retirements

The number of technologies and infrastructures present in zone z may change each planning period due to investments and retirements. These are given by Eqs. (11)–(14) for wind turbines, commercial conversion technologies, storage technologies and transport infrastructures, respectively.

$$N^{\mathrm{W}}_{zy} = N^{\mathrm{W}}_{z,y-1} + NI^{\mathrm{W}}_{zy} - NR^{\mathrm{W}}_{zy} \quad \forall\, z \in \mathbb{Z}, y \in \mathbb{Y} \tag{11}$$

The parameter N^{EW}_{zy}, in Eq. (6), accounts for the estimated retirement dates of existing wind turbines, thus Eq. (11) does not include any terms for existing wind turbines. See Eqs. (4) and (6) for how the generation from existing wind turbines is considered in the model.

$$N^{\mathrm{PC}}_{pzy} = \begin{cases} N^{\mathrm{EPC}}_{pz} + NI^{\mathrm{PC}}_{pzy} - NR^{\mathrm{PC}}_{pzy} - NR^{\mathrm{EPC}}_{pzy} & \forall\, p \in \mathbb{P}^{\mathrm{C}}, z \in \mathbb{Z}, y = 1 \\ N^{\mathrm{PC}}_{pz,y-1} + NI^{\mathrm{PC}}_{pzy} - NR^{\mathrm{PC}}_{pzy} - NR^{\mathrm{EPC}}_{pzy} & \forall\, p \in \mathbb{P}^{\mathrm{C}}, z \in \mathbb{Z}, y > 1 \end{cases} \tag{12}$$

$$N^{\mathrm{S}}_{szy} = \begin{cases} N^{\mathrm{ES}}_{sz} + NI^{\mathrm{S}}_{szy} - NR^{\mathrm{S}}_{szy} - NR^{\mathrm{ES}}_{szy} & \forall\, s \in \mathbb{S}, z \in \mathbb{Z}, y = 1 \\ N^{\mathrm{S}}_{sz,y-1} + NI^{\mathrm{S}}_{szy} - NR^{\mathrm{S}}_{szy} - NR^{\mathrm{ES}}_{szy} & \forall\, s \in \mathbb{S}, z \in \mathbb{Z}, y > 1 \end{cases} \tag{13}$$

$$N^{\mathrm{B}}_{bzz'y} = \begin{cases} N^{\mathrm{EB}}_{bzz'} + NI^{\mathrm{B}}_{bzz'y} & \forall\, b \in \mathbb{B}, z, z' \in \mathbb{Z}, y = 1 \\ N^{B}_{bzz'y-1} + NI^{\mathrm{B}}_{bzz'y} & \forall\, b \in \mathbb{B}, z, z' \in \mathbb{Z}, y > 1 \end{cases} \tag{14}$$

N^{W}_{zy}, N^{PC}_{pzy}, N^{S}_{szy} and $N^{\mathrm{B}}_{bzz'y}$ represent the total number of wind turbines, commercial conversion technologies, storage technologies in zone z and transport infrastructures connecting zones z and z'; N^{EPC}_{pz}, N^{ES}_{sz} and $N^{\mathrm{EB}}_{bzz'}$ are the number of existing commercial conversion technologies, existing storage technologies and existing transport infrastructure connections; NI^{W}_{zy}, NI^{PC}_{pzy}, NI^{S}_{szy} and $NI^{\mathrm{B}}_{bzz'y}$ indicate the additional technologies and infrastructures invested in at the beginning of planning period y; NR^{EPC}_{pzy} and NR^{ES}_{szy} are the number of existing commercial conversion technologies and existing storage technologies retired in planning period y; and NR^{W}_{zy}, NR^{PC}_{pzy}, and NR^{S}_{szy} are the number of wind turbines, commercial conversion and storage technologies retired at the beginning of planning period y, defined by Eqs. (15)–(17), respectively.

$$NR^{\mathrm{W}}_{zy} = \sum_{y'} RT^{\mathrm{W}}_{y'y} NI^{\mathrm{WT}}_{zy'} \quad \forall\, z \in \mathbb{Z}, y \in \mathbb{Y} \tag{15}$$

$$NR^{\mathrm{PC}}_{pzy} = \sum_{y'} RF^{P}_{py'y} NI^{P}_{pzy'} \quad \forall\, p \in \mathbb{P}^{\mathrm{C}}, z \in \mathbb{Z}, y \in \mathbb{Y} \tag{16}$$

$$NR^{\mathrm{S}}_{szy} = \sum_{y'} RF^{S}_{sy'y} NI^{S}_{szy'} \quad \forall\, s \in \mathbb{S}, z \in \mathbb{Z}, y \in \mathbb{Y} \tag{17}$$

$RT^{\mathrm{W}}_{y'y}$, $RF^{P}_{py'y}$ and $RF^{S}_{sy'y}$ are binary parameters with a value of 1 if the technologies invested in at the beginning of planning period y' retire at the beginning of planning period y; 0 otherwise. To illustrate this, Table 1 gives the retirement factors for a technology with a technical lifetime of 20 years (assuming that y is a 5-year planning interval).

The technical lifetime of transport infrastructures typically extends beyond the time planning horizon (e.g. 2050), therefore Eq. (14) does not contain the term for the retirements. If this is not the case, a similar approach to that for conversion and storage technologies can be followed to model the retirement of infrastructures.

5.3. Resource conversion

The net rate of production (or consumption) of resource r is:

$$P_{rzhdty} = \sum_{p} \mathscr{P}_{pzhdty} \alpha_{rpy} \quad \forall\, r \in \mathbb{R}, z \in \mathbb{Z}, h \in \mathbb{H}, d \in \mathbb{D}, t \in \mathbb{T}, y \in \mathbb{Y} \tag{18}$$

The rate of operation of a single commercial conversion technology p is bounded by its minimum and maximum capacities:

$$N^{\mathrm{PC}}_{pzy} p^{\min}_{p} \leqslant \mathscr{P}_{pzhdty} \leqslant N^{\mathrm{PC}}_{pzy} p^{\max}_{p}$$
$$\forall\, p \in \mathbb{P}^{\mathrm{C}}, z \in \mathbb{Z}, h \in \mathbb{H}, d \in \mathbb{D}, t \in \mathbb{T}, y \in \mathbb{Y} \tag{19}$$

The constraint on the maximum total number of commercial technologies that can be built in planning period y (i.e. the build rate) is:

$$\sum_{z} N^{\mathrm{PC}}_{pzy} \leqslant BR_{py} \quad \forall\, p \in \mathbb{P}^{\mathrm{C}}, y \in \mathbb{Y} \tag{20}$$

For domestic technologies, because their scale is much smaller than commercial technologies, the model tracks the numbers of domestic conversion technologies using a continuous variable, N^{PD}_{pzy}, which represents millions of technologies installed in each zone. The total production rate of all domestic technologies $p \in \mathbb{P}^{\mathrm{D}}$ in zone z is therefore:

$$\mathscr{P}_{pzhdty} \leq 10^{6} N^{\mathrm{PD}}_{pzy} p^{\max}_{p} \quad \forall\, p \in \mathbb{P}^{\mathrm{D}}, z \in \mathbb{Z}, h \in \mathbb{H}, d \in \mathbb{D}, t \in \mathbb{T}, y \in \mathbb{Y} \tag{21}$$

5.4. Resource storage

The net rate of utilisation of resource r from storage:

$$S_{rzhdty} = \sum_{s} \left(\mathscr{S}^{\mathrm{put}}_{szhdty} \sigma^{\mathrm{put}}_{sr,\mathrm{src},y} + \mathscr{S}^{\mathrm{hold}}_{szhdty} \sigma^{\mathrm{hold}}_{sr,\mathrm{dst},y} + \mathscr{S}^{\mathrm{get}}_{szhdty} \sigma^{\mathrm{get}}_{sr,\mathrm{dst},y}\right)$$
$$\forall\, r \in \mathbb{R}, z \in \mathbb{Z}, h \in \mathbb{H}, d \in \mathbb{D}, t \in \mathbb{T}, y \in \mathbb{Y} \tag{22}$$

The term $\sigma^{\star}_{srfy}$ ($\star \in$ {put, hold, get}) represents the conversion factor for each storage task, which can be used to define the input and output resources for each task, as well as the resource requirements and losses. The index f represents the source or destination of the task: for the put task, the source is the zone and the destination is the store; for the hold and the get tasks, the source is the store and the destination is the zone. The conversion factor is either positive or negative depending on whether the resource is added to or removed from the source or destination.

The maximum charging rate (or injectability) of storage facility s:

$$\mathscr{S}^{\mathrm{put}}_{szhdty} \leqslant N^{\mathrm{S}}_{szy} s^{\mathrm{put,max}}_{s} a_{sz} \quad \forall\, s \in \mathbb{S}, z \in \mathbb{Z}, h \in \mathbb{H}, d \in \mathbb{D}, t \in \mathbb{T}, y \in \mathbb{Y} \tag{23}$$

The maximum discharging rate (or deliverability) of storage facility s:

$$\mathscr{S}^{\mathrm{get}}_{szhdty} \leqslant N^{\mathrm{S}}_{szy} s^{\mathrm{get,max}}_{s} a_{sz} \quad \forall\, s \in \mathbb{S}, z \in \mathbb{Z}, h \in \mathbb{H}, d \in \mathbb{D}, t \in \mathbb{T}, y \in \mathbb{Y} \tag{24}$$

The inventory balance for storage facility s:

Table 1
Retirement factors for a technology with a lifetime of 20 years.

Investment period, y'	Retirement period, y						
	2015–2019	2020–2024	2025–2029	2030–2034	2035–2039	2040–2044	2045–2050
2015–2019	0	0	0	0	1	0	0
2020–2024	0	0	0	0	0	1	0
2025–2029	0	0	0	0	0	0	1
2030–2034	0	0	0	0	0	0	0
2035–2039	0	0	0	0	0	0	0
2040–2044	0	0	0	0	0	0	0
2045–2050	0	0	0	0	0	0	0

$$I_{szhdty} = n_h^{\text{hd}} \sum_r (\mathscr{S}^{\text{put}}_{szhdty}\sigma^{\text{put}}_{sr,\text{dst},y} + \mathscr{S}^{\text{hold}}_{szhdty}\sigma^{\text{hold}}_{sr,\text{src},y} + \mathscr{S}^{\text{get}}_{szhdty}\sigma^{\text{get}}_{sr,\text{src},y}) \quad \forall\, s \in \mathbb{S}, z \in \mathbb{Z}, h \in \mathbb{H}, d \in \mathbb{D}, t \in \mathbb{T}, y \in \mathbb{Y} \tag{25}$$

Eqs. (26) and (27) define the rate of the operation of the "hold" task as the inventory level from the last time interval divided by the length of the time interval:

$$\mathscr{S}^{\text{hold}}_{sz,1,dty} = I^{0,\text{sim}}_{szdty}/n_1^{\text{hd}} \quad \forall\, s \in \mathbb{S}, z \in \mathbb{Z}, d \in \mathbb{D}, t \in \mathbb{T}, y \in \mathbb{Y} \tag{26}$$

$$\mathscr{S}^{\text{hold}}_{szhdty} = I_{sz,h-1,dty}/n_h^{\text{hd}} \quad \forall\, s \in \mathbb{S}, z \in \mathbb{Z}, h > 1 \in \mathbb{H}, d \in \mathbb{D}, t \in \mathbb{T}, y \in \mathbb{Y} \tag{27}$$

The difference between the inventory at the end of the last hourly interval of the day, $|\mathbb{H}|$, and the inventory at the beginning of the first interval of the day is the surplus over that day type:

$$\delta^{\text{d}}_{szdty} = I_{sz,|\mathbb{H}|,dty} - I^{0,\text{sim}}_{szdty} \quad \forall\, s \in \mathbb{S}, z \in \mathbb{Z}, d \in \mathbb{D}, t \in \mathbb{T}, y \in \mathbb{Y} \tag{28}$$

Multiplying this by the number of times that day type, d, occurs in a week and then summing over d gives the surplus over one week:

$$\delta^{\text{t}}_{szty} = \sum_d \delta^{\text{d}}_{szdty} n_d^{\text{dw}} \quad \forall\, s \in \mathbb{S}, z \in \mathbb{Z}, t \in \mathbb{T}, y \in \mathbb{Y} \tag{29}$$

Similarly, multiplying the weekly surplus by the number of weeks in a season gives the surplus over a season. Summing these over all seasons yields the surplus over a year:

$$\delta^{\text{y}}_{szy} = \sum_t \delta^{\text{t}}_{szty} n_t^{\text{wt}} \quad \forall\, s \in \mathbb{S}, z \in \mathbb{Z}, y \in \mathbb{Y} \tag{30}$$

Eq. (31) is an optional cyclic constraint that can be applied to ensure that there is no accumulation of inventory in storage facility s over a year.

$$\delta^{\text{y}}_{szy} = 0 \quad \forall\, s \in \mathbb{S}, z \in \mathbb{Z}, y \in \mathbb{Y} \tag{31}$$

The variable $I^{0,\text{sim}}_{szdty}$ in Eqs. (26) and (28) is the initial inventory at the start of the particular day that gives rise to a profile with inventory levels that are the average levels over all identical days of that day type d, weeks in season t and years of yearly period y. Only this one profile (for each d, t and y) need be considered in the model because the costs and resource requirements for storage over all days, weeks and years will be the same as this "average" profile multiplied by the number of repeated days, week and year. If $I^{0,\text{act}}_{szdty}$ is the initial inventory for the first occurrence of day type d and the first week of season t, then the initial inventory for this included (or "simulated") profile is given by:

$$I^{0,\text{sim}}_{szdty} = I^{0,\text{act}}_{szdty} + [(n_d^{\text{dw}}-1)\delta^{\text{d}}_{szdty} + (n_t^{\text{wt}}-1)\delta^{\text{t}}_{szty} + (n_y^{\text{yy}}-1)\delta^{\text{y}}_{szy}]/2 \quad \forall\, s \in \mathbb{S}, z \in \mathbb{Z}, d \in \mathbb{D}, t \in \mathbb{T}, y \in \mathbb{Y} \tag{32}$$

The three terms in the square brackets respectively shift the initial inventory level from the first day of day type d up to the average level over all days of that day type, from the first week up to the average level over all weeks in season t and from the first year to the average over all years in yearly interval y. Note that the inventory levels over each hour are still distinct and correctly calculated according to the rates of addition and withdrawal of resource to and from the storage device (as well as losses, such as charge in a battery decaying). These relationships were explained in more detail in Samsatli et al. 2016 [5] and derived in the Appendix of Samsatli and Samsatli 2015 [3].

Eqs. (33)–(35) relate the inventories between day types, seasons and years, respectively:

$$I^{0,\text{act}}_{szdty} = I^{0,\text{act}}_{sz,d-1,ty} + n_{d-1}^{\text{dw}}\delta^{\text{d}}_{sz,d-1,ty} \quad \forall\, s \in \mathbb{S}, z \in \mathbb{Z}, d > 1 \in \mathbb{D}, t \in \mathbb{T}, y \in \mathbb{Y} \tag{33}$$

$$I^{0,\text{act}}_{sz,1,ty} = I^{0,\text{act}}_{sz,1,t-1,y} + n_{t-1}^{\text{wt}}\delta^{\text{t}}_{sz,t-1,y} \quad \forall\, s \in \mathbb{S}, z \in \mathbb{Z}, t > 1 \in \mathbb{T}, y \in \mathbb{Y} \tag{34}$$

$$I^{0,\text{act}}_{sz,1,1,y} = I^{0,\text{act}}_{sz,1,1,y-1} + n_{y-1}^{\text{yy}}\delta^{\text{y}}_{sz,y-1} \quad \forall\, s \in \mathbb{S}, z \in \mathbb{Z}, y > 1 \in \mathbb{Y} \tag{35}$$

The next set of constraints ensures that the inventory does not exceed the maximum storage capacity, $s_s^{\text{hold,max}}$, or go below the minimum storage capacity, $s_s^{\text{hold,min}}$, at all times. Because the entire time horizon is constructed from repeated profiles, the inventory increases or decreases by the same amount, $\delta^{\text{d}}_{szdty}$, each repeated day in day type d; by the same amount, δ^{t}_{szty}, each repeated week in season t; and by the same amount, δ^{y}_{szy}, each repeated year in yearly period y. Therefore, it is only necessary to constrain the hourly inventories in the first and last day of each day type, the first and last week of each season and the first and last year of each yearly interval. This is described by Eq. (36), which is a shorthand for the 16 sets of constraints formed by using either a positive or negative sign for each of the $\pm$ symbols (8 combinations of + or − for the three terms, multiplied by 2 for the two constraints: lower and upper bound).

$$s_s^{\text{hold,min}} N^{\text{S}}_{szy} a_{sz} \leqslant I_{szhdty} \pm [(n_d^{\text{dw}}-1)\delta^{\text{d}}_{szdty} \pm (n_t^{\text{wt}}-1)\delta^{\text{t}}_{szty} \pm (n_y^{\text{yy}}-1)\delta^{\text{y}}_{szy}]/2 \leqslant s_s^{\text{hold,max}} N^{\text{S}}_{szy} a_{sz} \quad \forall\, s \in \mathbb{S}, z \in \mathbb{Z}, h \in \mathbb{H}, d \in \mathbb{D}, t \in \mathbb{T}, y \in \mathbb{Y} \tag{36}$$

Finally, specific to the case study presented in this paper, compressors and expanders were specifically sized for and paired with each storage device. Eqs. (37) and (38) show the two constraints that apply for the large hydrogen storage tank, CGH_2S−L. Similar constraints are written for each storage device and its matching compressor and expander. These constraints allow compressors and expanders to be installed only if a matching storage device is installed but, for example, a compressor need not be installed if the storage device is always charged from a technology providing hydrogen at 20 MPa, hence the inequalities.

$$N^P_{\text{COMP}-\text{L},z} \leqslant N^S_{\text{CGH2S}-\text{L},z} \quad \forall\, z \in \mathbb{Z} \tag{37}$$

$$N^P_{\text{EXP}-\text{L},z} \leqslant N^S_{\text{CGH2S}-\text{L},z} \quad \forall\, z \in \mathbb{Z} \tag{38}$$

5.5. Resource transport

The net rate of transport of resource r into zone z from other zones is the difference between the incoming and outgoing flow rates:

$$Q_{rzhdty} = \sum_{z'|\nu_{z'z}=1} \sum_{l\in\mathbb{L}} [(\overline{\tau}_{lr,\mathrm{dst},y} + \hat{\tau}_{lr,\mathrm{dst},y} d_{z'z}) \mathcal{Q}_{lz'zhdty}] + \sum_{z'|\nu_{zz'}=1} \sum_{l\in\mathbb{L}} [(\overline{\tau}_{lr,\mathrm{src},y} + \hat{\tau}_{lr,\mathrm{src},y} d_{zz'}) \mathcal{Q}_{lzz'hdty}] \quad \forall\, r\in\mathbb{R}, z\in\mathbb{Z}, h\in\mathbb{H}, d\in\mathbb{D}, t\in\mathbb{T}, y\in\mathbb{Y} \tag{39}$$

The distance between the centres of demands of two zones is:

$$d_{zz'} = \sqrt{(x_z - x_{z'})^2 + (y_z - y_{z'})^2} \quad \forall\, z, z'\in\mathbb{Z} \tag{40}$$

The operation of transport technology l cannot exceed its maximum rate:

$$\mathcal{Q}_{lzz'hdty} \leqslant \sum_{b\in\mathbb{B}} q_l^{\max} N^{\mathrm{B}}_{bzz'y}|_{LB_{lb}=1\wedge\nu_{zz'}=1} \quad \forall\, l\in\mathbb{L}; z, z'\in\mathbb{Z}; h\in\mathbb{H}, d\in\mathbb{D}, t\in\mathbb{T}, y\in\mathbb{Y} \tag{41}$$

The capacity of infrastructure b cannot be exceeded by the rate of operation of all transport technologies utilising the infrastructure:

$$\sum_{l\in\mathbb{L}} \mathcal{Q}_{lzz'hdty} LB_{lb} \leqslant b_b^{\max} N^{\mathrm{B}}_{bzz'y} \quad \forall\, b\in\mathbb{B}; z, z'\in\mathbb{Z}; h\in\mathbb{H}, d\in\mathbb{D}, t\in\mathbb{T}, y\in\mathbb{Y} \tag{42}$$

Finally, bidirectional and unidirectional connections can be modelled by setting the parameter β_b: if $\beta_b = -1$ then the connection is unidirectional and can only be built in one direction (e.g. pipelines for CO_2); if $\beta_b = 0$ then connections are unidirectional but two separate connections can be made in opposite directions (e.g. railways, roads) – each connection is treated separately and so making a bidirectional connection in this way costs twice as much as a single connection; and if $\beta_b = 1$ then only a single bi-directional connection can be made (e.g. pipelines for natural gas, hydrogen, etc. and electricity transmission lines). Eqs. (43) and (44) give the constraints for bidirectional and unidirectional infrastructures, respectively.

$$N^{\mathrm{B}}_{bzz'y} = N^{\mathrm{B}}_{bz'zy} \quad \forall\, b\in\mathbb{B}|\beta_b = 1, z \neq z'\in\mathbb{Z} \tag{43}$$

$$N^{\mathrm{B}}_{bzz'y} + N^{\mathrm{B}}_{bz'zy} \leq 1 \quad \forall\, b\in\mathbb{B}|\beta_b = -1, z \neq z'\in\mathbb{Z} \tag{44}$$

5.6. Resource import and export

The maximum rates of import and export of resource r in and out of zone z, respectively, are:

$$M_{rzhdty} \leqslant m^{\max}_{rzhdty} \quad \forall\, r\in\mathbb{R}, z\in\mathbb{Z}, h\in\mathbb{H}, d\in\mathbb{D}, t\in\mathbb{T}, y\in\mathbb{Y} \tag{45}$$

$$X_{rzhdty} \leqslant \chi^{\max}_{rzhdty} \quad \forall\, r\in\mathbb{R}, z\in\mathbb{Z}, h\in\mathbb{H}, d\in\mathbb{D}, t\in\mathbb{T}, y\in\mathbb{Y} \tag{46}$$

In the case studies, import of natural gas through the different interconnectors is considered. The maximum import rate, $m^{\max}_{\mathrm{NG},zhdty}$, was also obtained from the National Grid's gas transmission operational data [31].

5.7. Objective function

The objective function is the *minimisation* of Z, which is a weighted sum of all of the impacts of all activities associated with the provision of energy and the total energy production:

$$Z = \sum_{iy} \omega_i (\mathscr{I}^{\mathrm{W}}_{iy} + \mathscr{I}^{\mathrm{P}}_{iy} + \mathscr{I}^{\mathrm{S}}_{iy} + \mathscr{I}^{\mathrm{Q}}_{iy} + \mathscr{I}^{\mathrm{w}}_{iy} + \mathscr{I}^{\mathrm{fp}}_{iy} + \mathscr{I}^{\mathrm{fs}}_{iy} + \mathscr{I}^{\mathrm{fq}}_{iy} + \mathscr{I}^{\mathrm{vp}}_{iy} + \mathscr{I}^{\mathrm{vs}}_{iy} + \mathscr{I}^{\mathrm{vq}}_{iy} + \mathscr{I}^{\mathrm{m}}_{iy} + \mathscr{I}^{\mathrm{x}}_{iy} + \mathscr{I}^{\mathrm{U}}_{iy} - \mathscr{I}^{\mathrm{Rev}}_{iy}) - \epsilon \sum_{y} E^{\mathrm{TOT}}_y n^{yy}_y \tag{47}$$

where the index i is an element of the set of key performance indicators $\mathbb{I} \equiv \{\mathrm{Cost}, CO_2\}$. ω_i and ϵ are weighting factors that enable different objective functions to be defined, for example:

- Maximise profit: set $\omega_{\mathrm{Cost}} = 1, \omega_{CO_2} = 0, \epsilon = 0, V_{riy} \neq 0, D^{\mathrm{comp}}_{rzhdty} = D^{\mathrm{act}}_{rzhdty}$ and $D^{\mathrm{opt}}_{rzhdty} = 0$;
- Minimise cost: set $\omega_{\mathrm{Cost}} = 1, \omega_{CO_2} = 0, \epsilon = 0, V_{riy} = 0, D^{\mathrm{comp}}_{rzhdty} = D^{\mathrm{act}}_{rzhdty}$ and $D^{\mathrm{opt}}_{rzhdty} = 0$;
- Minimise CO_2 emissions: set $\omega_{\mathrm{Cost}} = 0, \omega_{CO_2} = 1, \epsilon = 0, D^{\mathrm{comp}}_{rzhdty} = D^{\mathrm{act}}_{rzhdty}$ and $D^{\mathrm{opt}}_{rzhdty} = 0$;
- Maximise total energy production: set $\omega_i = 0$, $\epsilon = 1$, $D^{\mathrm{comp}}_{rzhdty} = 0$ and $D^{\mathrm{opt}}_{rzhdty} = D^{\mathrm{act}}_{rzhdty}$.

The terms on the right-hand-side of Eq. (47) are defined as follows:

- Total net present capital impact for new wind turbines:

$$\mathscr{I}^{\mathrm{W}}_{iy} = \varsigma D^{\mathrm{W}}_{iy} C^{\mathrm{W}}_{iy} \sum_{z} NI^{\mathrm{W}}_{zy} \quad \forall\, i\in\mathbb{I}, y\in\mathbb{Y} \tag{48}$$

- Total net present capital impact for building new production technologies:

$$\mathscr{I}^{\mathrm{P}}_{iy} = \varsigma \sum_{pz} D^{\mathrm{C}}_{piy} C^{\mathrm{P}}_{piy} NI^{\mathrm{PC}}_{pzy} \quad \forall\, i\in\mathbb{I}, y\in\mathbb{Y} \tag{49}$$

- Total net present capital impact for building new storage facilities:

$$\mathscr{I}^{\mathrm{S}}_{iy} = \varsigma \sum_{sz} D^{\mathrm{C}}_{siy} C^{\mathrm{S}}_{siy} NI^{\mathrm{S}}_{szy} \quad \forall\, i\in\mathbb{I}, y\in\mathbb{Y} \tag{50}$$

- Total net present capital impact for building new transport infrastructures:

$$\mathscr{I}^{\mathrm{Q}}_{iy} = \varsigma \sum_{bzz'} D^{\mathrm{C}}_{biy} C^{\mathrm{B}}_{biy} NI^{\mathrm{B}}_{bzz'y} d_{zz'} (1 - 0.5|_{\beta_b=1}) \quad \forall\, i\in\mathbb{I}, y\in\mathbb{Y} \tag{51}$$

- Total net present O&M impact for wind turbines:

$$\mathscr{I}^{\mathrm{w}}_{iy} = \varsigma D^{\mathrm{OM}}_{y} \phi^{\mathrm{W}}_{iy} \sum_{z} (N^{\mathrm{W}}_{zy} + NU^{\mathrm{EW}}_{zy}) \quad \forall\, i\in\mathbb{I}, y\in\mathbb{Y} \tag{52}$$

- Total net present fixed O&M impact of production technologies:

$$\mathscr{I}^{\mathrm{fp}}_{iy} = \varsigma D^{\mathrm{OM}}_{y} \sum_{pz} \phi^{\mathrm{P}}_{piy} N^{\mathrm{PC}}_{pzy} \quad \forall\, i\in\mathbb{I}, y\in\mathbb{Y} \tag{53}$$

- Total net present fixed O&M impact of storage facilities:

$$\mathscr{I}^{\mathrm{fs}}_{iy} = \varsigma D^{\mathrm{OM}}_{y} \sum_{sz} \phi^{\mathrm{S}}_{siy} N^{\mathrm{S}}_{szy} \quad \forall\, i\in\mathbb{I}, y\in\mathbb{Y} \tag{54}$$

- Total net present fixed O&M impact of transport infrastructures:

$$\mathscr{I}^{\mathrm{fq}}_{iy} = \varsigma D^{\mathrm{OM}}_{y} \sum_{bzz'} \phi^{\mathrm{B}}_{biy} N^{\mathrm{B}}_{bzz'y} d_{zz'} (1 - 0.5|_{\beta_b=1}) \quad \forall\, i\in\mathbb{I}, y\in\mathbb{Y} \tag{55}$$

- Total net present variable operating impact of production facilities:

$$\mathscr{I}^{\text{vp}}_{iy} = \varsigma D^{\text{OM}}_{y} \sum_{pzhdt} \varphi^{\text{P}}_{piy} \mathscr{P}_{pzhdty} n^{\text{hd}}_{h} n^{\text{dw}}_{d} n^{\text{wt}}_{t} \quad \forall\, i \in \mathbb{I}, y \in \mathbb{Y} \tag{56}$$

Note that the operating impact for a particular technology does not directly include the impact of any raw materials consumed. The resource balance ensures that consumption of a resource must be balanced by: import, transport from another zone, production by other technologies, utilisation of available resource (e.g. wind, biomass) or utilisation of stored resource. Each of these routes to the provision of the raw materials needed for the operation of a technology incurs its own impacts, thus: import will have a direct impact proportional to the quantity imported; transport from another zone incurs transportation impacts in addition to the impacts of producing/importing/etc. that resource in the other zone; production by other technologies incurs the operating impacts of those technologies plus the same account of impacts for the feedstocks required by those technologies; utilisation of available resource incurs impacts; and utilisation of stored resource incurs direct impacts associated with retrieving the resource from storage as well as the impacts of producing and storing that resource at an earlier time. This is a system-wide approach to accounting for costs and emission rather than treating each instance of a technology as an individual business entity that must pay for all raw materials and receive revenue for any products sold. When calculating the total costs and revenues of all entities in the system, the costs incurred by the consumer of one resource will cancel with the revenue accrued by the producer(s) of that resource, so the two approaches are equivalent.

- Total net present variable operating impact of storage facilities:

$$\mathscr{I}^{\text{vs}}_{iy} = \varsigma D^{\text{OM}}_{y} \sum_{szhdt} (\varphi^{\text{SP}}_{siy} \mathscr{S}^{\text{put}}_{szhdty} + \varphi^{\text{SH}}_{siy} \mathscr{S}^{\text{hold}}_{szhdty} + \varphi^{\text{SG}}_{siy} \mathscr{S}^{\text{get}}_{szhdty}) n^{\text{hd}}_{h} n^{\text{dw}}_{d} n^{\text{wt}}_{t} \quad \forall\, i \in \mathbb{I}, y \in \mathbb{Y} \tag{57}$$

- Total net present variable operating impact of transport technologies:

$$\mathscr{I}^{\text{vq}}_{iy} = \varsigma D^{\text{OM}}_{y} \sum_{lzz'hdt} (\hat{\varphi}^{\text{Q}}_{liy} d_{zz'} + \overline{\varphi}^{\text{Q}}_{liy}) \mathscr{Q}_{lzz'hdty} n^{\text{hd}}_{h} n^{\text{dw}}_{d} n^{\text{wt}}_{t} \quad \forall\, i \in \mathbb{I}, y \in \mathbb{Y} \tag{58}$$

- Total net present impact from resource production:

$$\mathscr{I}^{\text{U}}_{iy} = \varsigma D^{\text{OM}}_{y} \left(\sum_{r \in \mathbb{R}-\mathbb{C}} \sum_{zhdt} c^{\text{U}}_{rihdty} U_{rzhdty} n^{\text{hd}}_{h} n^{\text{dw}}_{d} n^{\text{wt}}_{t} + \sum_{czt} c^{\text{Bio}}_{city} A^{\text{Bio}}_{czy} Y^{\text{Bio}}_{czty} \right) \quad \forall\, i \in \mathbb{I}, y \in \mathbb{Y} \tag{59}$$

- Total net present import impact:

$$\mathscr{I}^{\text{m}}_{iy} = \varsigma D^{\text{OM}}_{y} \sum_{rzhdt} c^{\text{M}}_{rihdty} M_{rzhdty} n^{\text{hd}}_{h} n^{\text{dw}}_{d} n^{\text{wt}}_{t} \quad \forall\, i \in \mathbb{I}, y \in \mathbb{Y} \tag{60}$$

- Total net present export impact:

$$\mathscr{I}^{\text{x}}_{iy} = \varsigma D^{\text{OM}}_{y} \sum_{rzhdt} c^{\text{X}}_{rihdty} X_{rzhdty} n^{\text{hd}}_{h} n^{\text{dw}}_{d} n^{\text{wt}}_{t} \quad \forall\, i \in \mathbb{I}, y \in \mathbb{Y} \tag{61}$$

- Total net present revenue from the sales of energy services for satisfying demands:

$$\mathscr{I}^{\text{Rev}}_{iy} = \varsigma D^{\text{OM}}_{y} \sum_{rzhdt} V_{riy} (D^{\text{comp}}_{rzhdty} + D^{\text{sat}}_{rzhdty}) n^{\text{hd}}_{h} n^{\text{dw}}_{d} n^{\text{wt}}_{t} \quad \forall\, i \in \mathbb{I}, y \in \mathbb{Y} \tag{62}$$

- Total energy production:

$$E^{\text{TOT}}_{y} = \sum_{r \in \mathbb{E}} \sum_{pzhdt} \mathscr{P}_{pzhdty} \alpha_{rpy} n^{\text{hd}}_{h} n^{\text{dw}}_{d} n^{\text{wt}}_{t} \quad \forall\, y \in \mathbb{Y} \tag{63}$$

The term $0.5|_{\beta_b=1}$ in Eqs. (51) and (55) only appears for bidirectional infrastructures, where $\beta_b = 1$, so that although $N^{\text{B}}_{bzz'y} = N^{\text{B}}_{bz'zy}$ (see Constraint (43)) the cost of connections in only one direction is incurred. If $\beta_b \neq 1$, then this term disappears and connections in each direction are costed independently.

The capital costs (i.e. when i = Cost) are calculated by assuming investments are financed over a given number of years $\lambda_\star$ – the economic life of the technology. When an investment is made in a technology in planning period y, the capital cost (at planning period y's prices) is multiplied by a factor to give the $\lambda_\star$ annual repayments that must be made based on a finance rate of γ; the first repayment is one year after receipt of the funds, which occurs at the start of the first year in the period. A second factor discounts all of these annual repayments back to the beginning of period y and a third factor discounts this figure back to the beginning of the first planning period; both use a discount rate of ι. The overall product of these factors is denoted by $D^{\text{C}}_{\star iy}$ and is given by Eq. (64).

$$D^{\text{C}}_{\star iy} = \begin{cases} (1+\iota)^{-5(y-1)} \left[\dfrac{\gamma(1+\gamma)^{\lambda_\star}}{(1+\gamma)^{\lambda_\star}-1} \right] \displaystyle\sum_{\tilde{y}=1}^{\lambda_\star} (1+\iota)^{-\tilde{y}} & \forall\, i = \text{Cost}, y \in \mathbb{Y} \\ 1 & \forall\, i \neq \text{Cost}, y \in \mathbb{Y} \end{cases} \tag{64}$$

Note that $\lambda_\star$ may be different for each individual technology (including any of the conversion, storage and infrastructure technologies), so $D^{\text{C}}_{\star iy}$ also depends on the technology. In Eqs. (49)–(51), the $\star$ subscript is replaced by p, s or b to indicate that the discounting factor relates respectively to conversion, storage or infrastructure technologies, calculated using λ_p, λ_s or λ_b. The equivalent factor for wind turbines is D^{W}_{iy}, using a separate economic life for wind turbines.

Operating and maintenance costs are assumed to be paid at the beginning of each year in any planning period y. These costs are also discounted back to the beginning of period y and back to the start of the first period. This is the same for all technologies, given by the parameter D^{OM}_{iy}.

$$D^{\text{OM}}_{iy} = \begin{cases} (1+\iota)^{-5(y-1)} \sum_{\tilde{y}=1}^{5} (1+\iota)^{1-\tilde{y}} & \forall\, i = \text{Cost}, y \in \mathbb{Y} \\ 5 & \forall\, i \neq \text{Cost}, y \in \mathbb{Y} \end{cases} \tag{65}$$

6. Great Britain case studies

The MILP model presented in Section 5 is used to examine different scenarios of satisfying the demands for energy services in GB. In each case study, the model simultaneously determines the design and operation of the integrated multi-vector networks that satisfy, in the most effective manner, the spatially-distributed and temporally-variable energy service demands utilising different primary resources, whose availability is also a function of space and time, in order to meet different objectives and constraints. The different technologies for conversion, transport and storage of resources illustrated in the network superstructure in Fig. 5 are considered. Each technology has 3 different sizes – small, medium and large – in order to capture the economies of scale and determine whether a centralised system is more effective than a distributed one, or vice versa. A centralised system will feature large technologies in a few strategic locations and a large transport network in the solution, whereas a distributed system will contain more small technologies and fewer interconnected zones (i.e. more self-sufficient zones). The model determines simultaneously the best combination of technologies to employ, where to locate them, when to invest in them and how to operate them together.

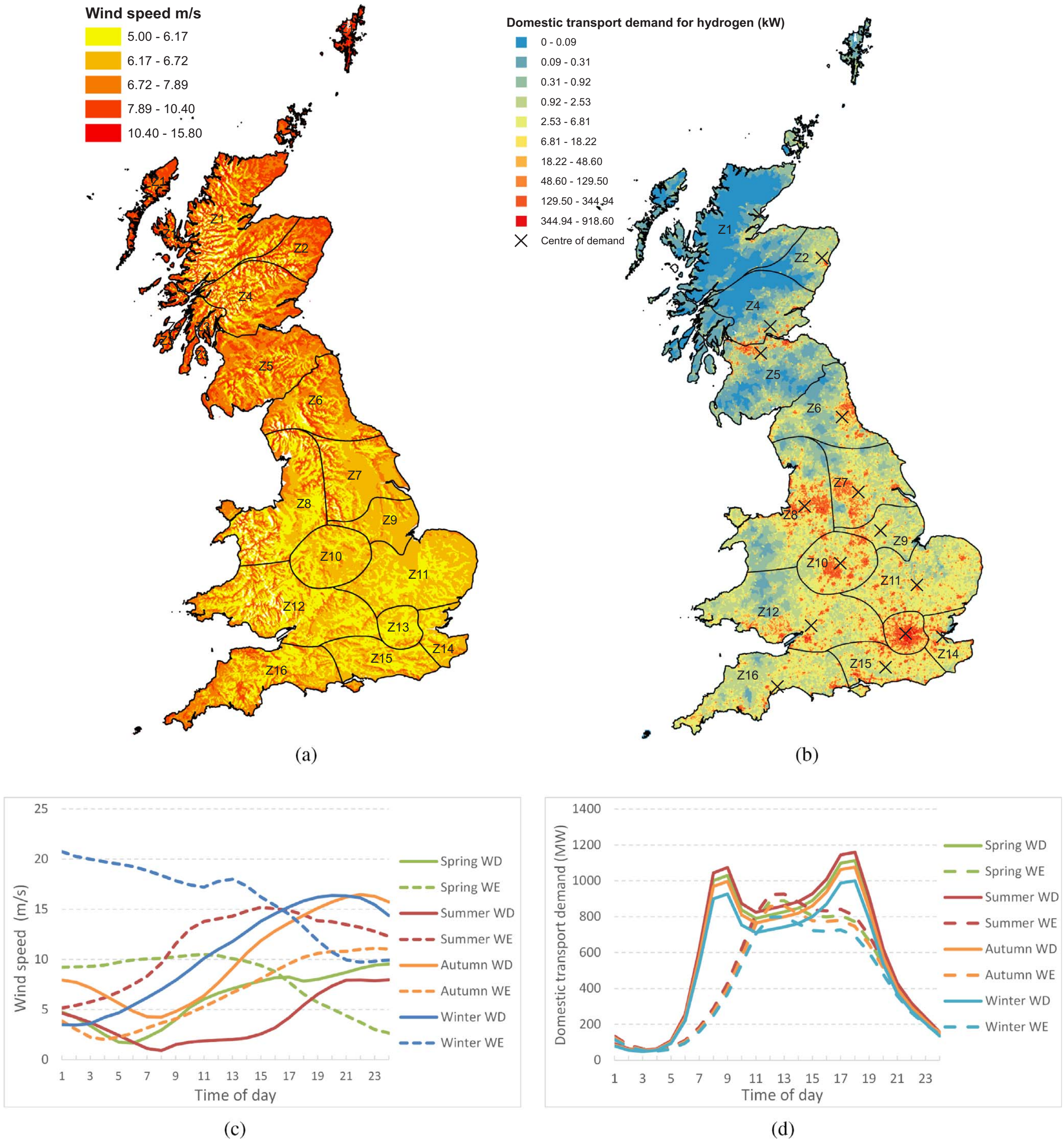


Fig. 6. Representative data for resource availability and demands used in the case studies: spatial distribution of (a) wind availability [32] and (b) domestic transport demands [33]; and temporal variability of (c) wind availability [34] and (d) domestic transport demands [35] for zone 13 (similar time-series profiles were derived for other zones).

The first case includes only a subset of the network superstructure and considers a very relevant problem: whether and how the domestic transport sector in GB can be decarbonised by 2050 by utilising only onshore wind power in an integrated electricity and hydrogen network – the objective function is the minimisation of the total net present cost subject to the available land area for wind turbines in each zone, which was determined by applying different technical and environmental constraints in a GIS analysis. The next 2 cases consider the entire network superstructure and determine the most effective way of meeting the domestic demands for electricity, heat and mobility utilising natural gas, wind power and biomass: one case maximises the total net present value and the other minimises the total CO_2 emissions, both subject to the maximum land area allowed for wind turbines and biomass production. The final case has an objective function of maximisation of energy production, with the aim of determining how much of each of the energy service demands can be satisfied given a certain amount of land area where wind turbines can be installed and biomass can be grown; this case does not include natural gas as a primary resource.

A discount factor of 3.5% and a finance rate of 8% were used in the case studies. The properties of all the technologies considered in the network superstructure are given in the Supplementary Material.

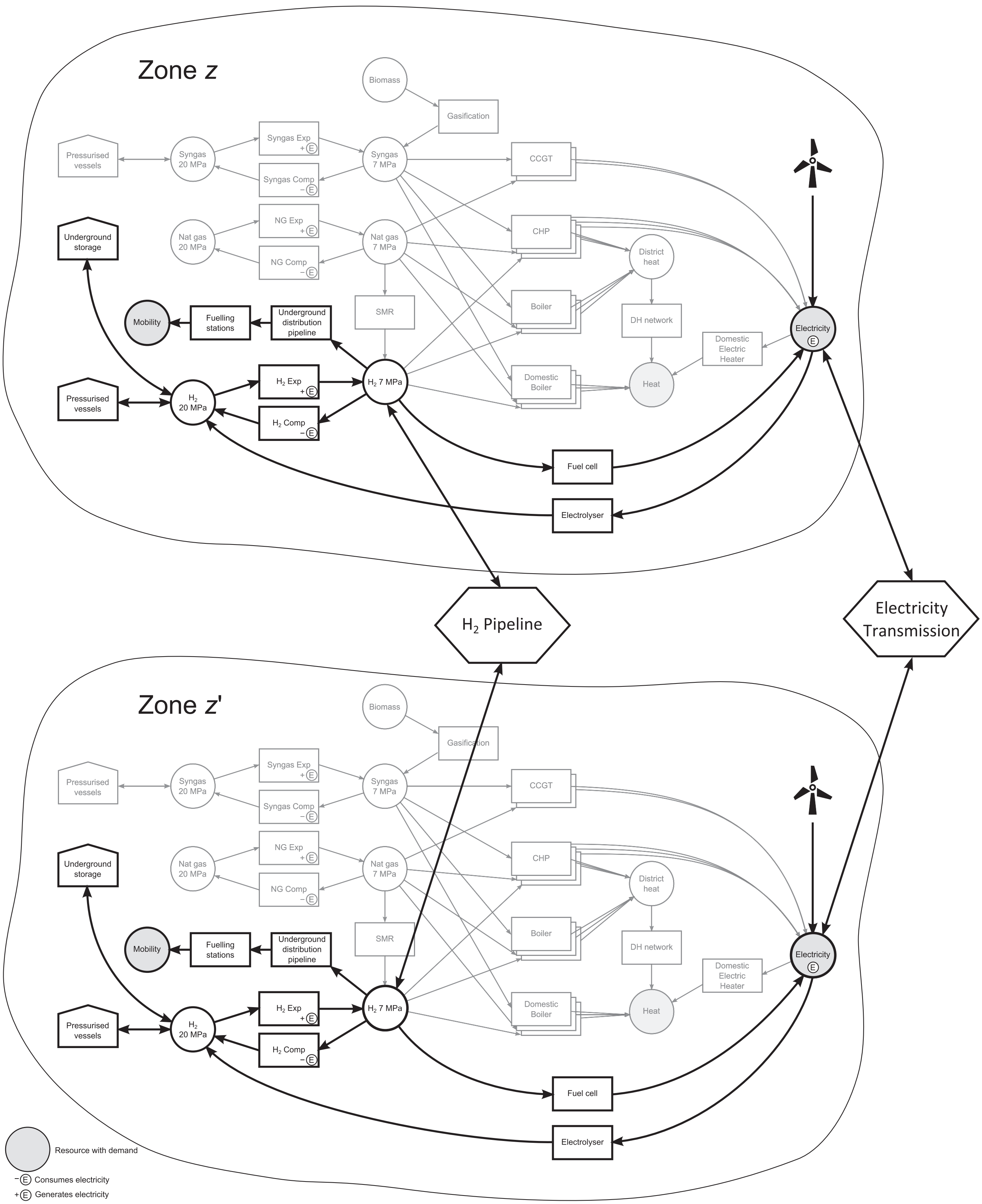


Fig. 7. Subset of the network superstructure (in bold outline): integrated wind-hydrogen-electricity network to decarbonise the domestic mobility sector.

6.1. Least cost scenario of meeting mobility demands using wind power only

The transport sector, which still depends almost exclusively on oil, is a very challenging sector to decarbonise. Hydrogen is a strong contender as a future transport fuel. Hydrogen's full environmental advantages can be achieved if it is produced from renewables and GB has a very good potential for wind power. In this first example, the aim is to determine whether and how all of the domestic transport demands

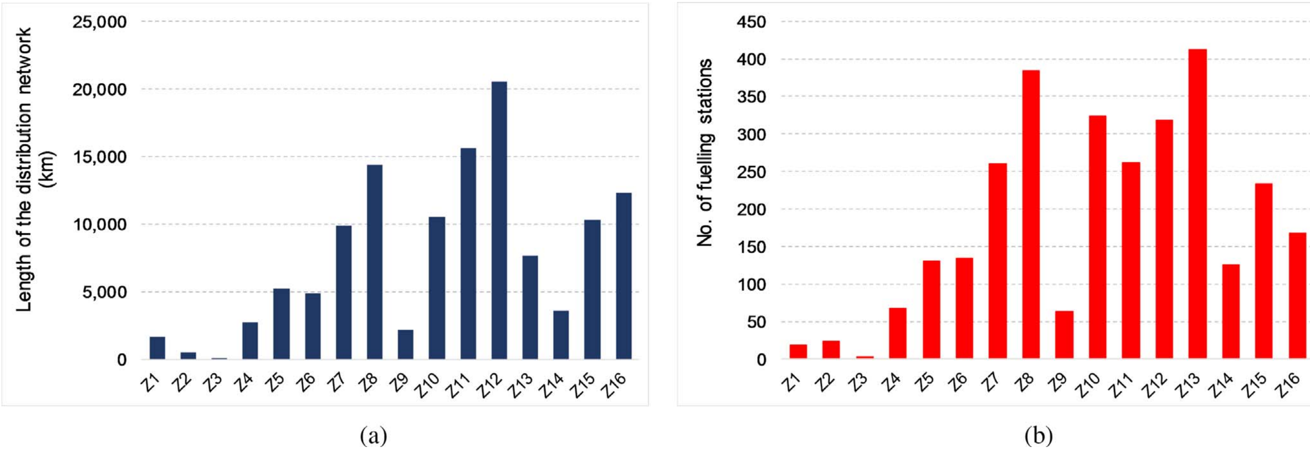


Fig. 8. Characteristics of the distribution networks for the case studies: (a) length of the hydrogen distribution pipeline; and (b) number of fuelling stations.

Table 2
Land footprint constraints for wind turbines [5].

Criteria used to determine the total land area in each zone suitable for siting wind turbines		GIS data used
1	Average wind speed of at least 5 m/s at 45 m above ground level	Wind speed from Department of Trade and Industry [32]
2	Slope of less than 15%	Slope from Ordnance Survey's Meridian 2 [38]
3	Access: a minimum distance of 500 m from minor road network for access	Minor roads from Ordnance Survey's Meridian 2 [38]
4	Connectivity to National Grid: at least 200 m but not more than 1500 m from road network	Minor and major roads from Ordnance Survey's Meridian 2 [38]
5	Not in Sites of Special Scientific Interest (SSSI)	SSSI from Natural England [39] and Natural Resources Wales [40]
6	Population impact: at least 500 m from developed land used area (DLUA)	DLUA from Ordnance Survey's Meridian 2 [38]
7	Water pollution: at least 200 m from river	Rivers from Ordnance Survey's Meridian 2 [38]
8	Wildlife and interference: at least 250 m from woodland	Woodlands from Ordnance Survey's Meridian 2 [38]
9	Safety: at least 5 km from airports	Airports from ShareGeo Open [41]
10	Not occupied by existing wind turbines including spacing between turbines of 5 rotor diameters	Existing wind turbines from the Virtual Wind Farm Model [34]

in GB can be satisfied, by 2050, by converting wind power to hydrogen for fuel cell vehicles. Given the spatio-temporal characteristics of wind availability and hydrogen demands in Fig. 6, the aim is to determine: how many wind turbines will be needed and what zones will they be located; whether to transmit the energy as electricity or hydrogen or both; the structure of transmission network; the conversion and storage technologies required, their locations and sizes. For this problem, only a subset of the network superstructure presented in Fig. 4 was considered – this is shown in Fig. 7. In this example, the existing electricity networks and wind turbines were not considered: it was assumed that their capacity is already committed to satisfying existing loads and therefore new ones will have to built in order to satisfy the hydrogen demands from fuel cell vehicles. Hydrogen underground storage facilities were considered in order to determine whether it is worth converting any of the existing natural gas underground storage to hydrogen.

For this problem, the cost of the distribution network is a significant part of the total cost. This was estimated from the length of the distribution pipeline networks and the number of the fuelling stations, both of which are functions of the demand density at the 1 km level, denoted by $D(x,y)$ in Eqs. (66) and (67) and presented in Fig. 6(b), and the capacity of fuelling stations, C, which for this study was taken to be 1,500 kg/day [36]. The length of the hydrogen distribution network and the number of fuelling stations in each zone, which are presented in Fig. 8, were determined by evaluating the surface integrals given by Eqs. (66) and (67).

$$L_z^{network} = \iint_{S_z} \frac{D(x,y)}{C} \sqrt{(x-x_z)^2 + (y-y_z)^2}\; dx\, dy \tag{66}$$

$$N_z^{stations} = \left\lceil \frac{1}{C} \iint_{S_z} D(x,y)\, dx\, dy \right\rceil \tag{67}$$

As one of the main concerns about wind turbines is their siting, a

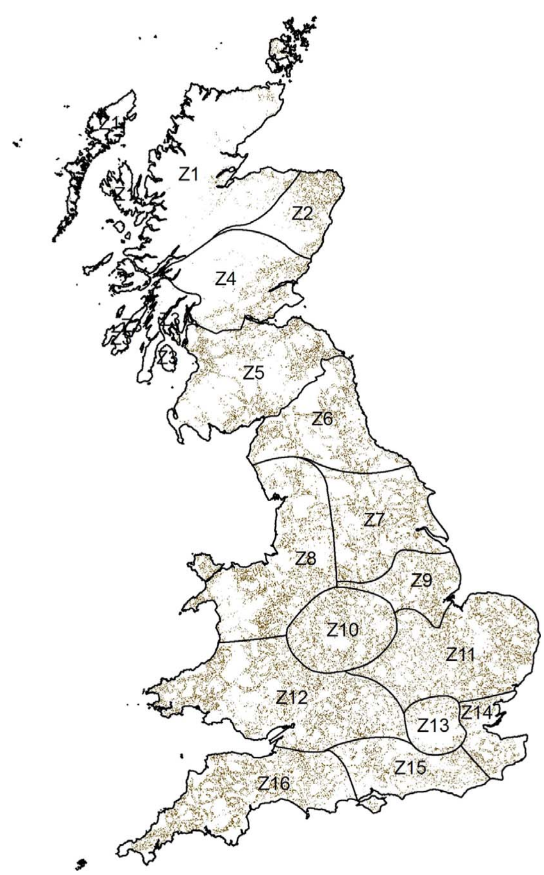


Fig. 9. Available land area after applying constraints 1–10 in Table 2.

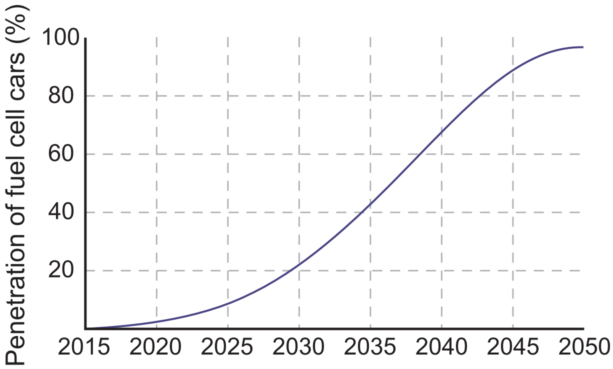


Fig. 10. Trajectory of penetration of fuel cell cars used in the case studies, based on the trajectory used by Almansoori and Shah [8].

GIS analysis was performed to determine the land area suitable for siting wind turbines. Table 2 gives the 10 constraints that need to be satisfied in order for an area of land to be considered suitable. The intersection of all of these constraints is represented by the shaded region in Fig. 9, which comprises 2% of the total land area in GB. The total available area in each zone defines the land footprint constraint in the model (see Eq. (7)).

The hourly time-series wind speed data for each zone were obtained from the Virtual Wind Farm Model [34] and the future wind speed data were derived using the UK Climate Projection 09 probabilistic projections of wind speed [37]. Similarly, the hourly demand time-series data were calculated from the Department for Transport data for vehicular usage [35] and the future demand data were projected assuming a trajectory of penetration of fuel cell cars shown in Fig. 10, which is based on the trajectory used by Almansoori and Shah [8].

Fig. 11 shows the gradual build up of technologies and infrastructures, from no hydrogen demand in 2015 to a fully decarbonised domestic transport sector in 2050, for the case where the objective is the minimisation of the net present cost (i.e. all cost were discounted back to 2015). It can be seen that both networks for electricity (blue and green connections) and hydrogen (red connections) are required in order to meet all of the hydrogen demands from fuel cell vehicles, as well as wind turbines, electrolysers and hydrogen storage in strategic locations. Wind turbines are installed predominantly in Scotland and in the north and south of England; these are always accompanied by electrolysers and hydrogen storage. There are no technologies installed in the east of England, where the hydrogen demands are satisfied by transporting hydrogen produced from other parts of GB, through the pipeline network. Although there are no wind turbines installed in south Wales, there are electrolysers and hydrogen storage; the hydrogen is produced locally using electricity transmitted from neighbouring zones via the HVAC OHL lines. As the hydrogen demands increase over the years, due to the increasing penetration of fuel cell vehicles, more wind turbines and technologies are installed and the transmission networks become larger and connected across the whole of GB.

It can also be seen in Fig. 11 that the Humbly Grove underground storage in zone 15, one of the four underground storage facilities considered in the case study, is deployed for hydrogen storage in 2035–2039, replacing some of the medium-sized pressurised vessels acquired in earlier periods. In period 2040–2044 all hydrogen storage vessels in zone 15 have retired. From its deployment in 2035–2039, the Humbly Grove underground storage plays an important role in balancing the hourly demand and supply for hydrogen of the zones in England. Fig. 12(a) presents the hourly inventory of hydrogen in this facility in the final period, 2045–2050, where its utilisation is at its highest level in autumn. Figs. 12(b) and 12(c) show a couple of snapshots of the operation of the hydrogen transmission network during weekdays in summer in 2045–2050: during one of the peak hours (e.g. at 08:00 h), the Humbly Grove underground storage supplies hydrogen to neighbouring zones via the transmission pipeline network but when the demands are low (e.g. at 22:00 h), hydrogen is transmitted back into zone 15 in order to replenish the inventory of hydrogen in this large-scale storage facility.

Fig. 13 gives the total capital and O&M costs incurred in every period for each component of the network; the net present cost of the entire network is £67.7 billion, £49bn of which is due to the distribution network, calculated assuming investments in each planning period are financed over 20 years. Therefore, the selling price of the renewable hydrogen will need to be £68/MWh (£2.27/kg) in order to break even. This network will result in 2 billion tonnes of *avoided* CO_2 emissions, which was determined based on the CO_2 emissions factor reported by GOV.UK [42] for the UK domestic transport sector and the fraction of penetration of fuel cell vehicles presented in Fig. 10.

6.2. Optimal scenarios of satisfying heat, electricity and mobility demands

The following scenarios consider the entire network superstructure presented in Fig. 5 and examine how to satisfy all of GB's domestic energy demands (i.e. heat, electricity and mobility) under different objectives: maximisation of net present value (profit), minimisation of CO_2 emissions and maximisation of energy production using only renewable primary sources.

The scenarios consider some of the existing assets in GB, which are shown in Fig. 14. They include the existing natural gas and electricity networks, the location of the natural gas terminals (which also indicate the local production of natural gas from the UK Continental Shelf) and imports through different interconnectors; existing wind turbine capacity; and existing CCGT plants. It was assumed that the natural gas availability decreases by 2% every year; the existing wind turbine capacity retires over 15 years (i.e. 1/3 of the capacity retires every 5 years); and the existing CCGT plants retire 30 years after they were built. The existing coal-fired power stations are not considered due their legislated closure in 2025 and the fact that coal generation in the UK has decreased significantly in recent years and now only accounts for less than 10% of the energy mix [43–45].

In the scenarios, Miscanthus was considered as the biomass resource. It is a perennial grass and a strong candidate as a feedstock for second-generation bioenergy refineries because of its high yield potential, low energy demands and low production costs [46], as well as its good performance on marginal and degraded land [47]. The following properties were used for Miscanthus: calorific value of 3.92 MWh/odt; yield potential of 5.34 odt/ha/season in winter and 3.58 odt/ha/season in spring (no harvesting takes place in summer and autumn, thus the overall yield is 8.92 odt/ha/yr); production cost of £41.59/odt; and CO_2 emissions of 58.75 $kgCO_2$/odt. It was assumed that Miscanthus will be cultivated on grassland. Fig. 15(a) presents the land areas in GB that are classified as grassland, according to the UK Land Cover Map 2007 [48], which is about 32% of the total land area. As was done with the wind turbines, a GIS analysis was performed to eliminate the land areas that are not suitable for production of biomass, based on the constraints presented in Refs. [4,49]. GIS is used to identify areas with elevation higher than 250 m, slope greater than 15%, urban areas, roads, rivers and parks, protected areas and Sites of Special Scientific Interest (SSSI), and areas of outstanding natural beauty. These polygons are intersected with the original areas of grassland and this intersection is then subtracted from the original areas to leave the areas of grassland that are suitable for biomass production. Fig. 15(b) shows the remaining land areas, which were used as upper bounds on the biomass land footprint constraints in each zone defined by Eqs. (9) and (10). The remaining land area after the screening is about 13% of the total GB land area but in the scenarios it was further imposed that only 10% of the suitable land area is the maximum allowable for biomass production ($f_y^{\mathrm{nat}} = 0.1$), as a conservative assumption.

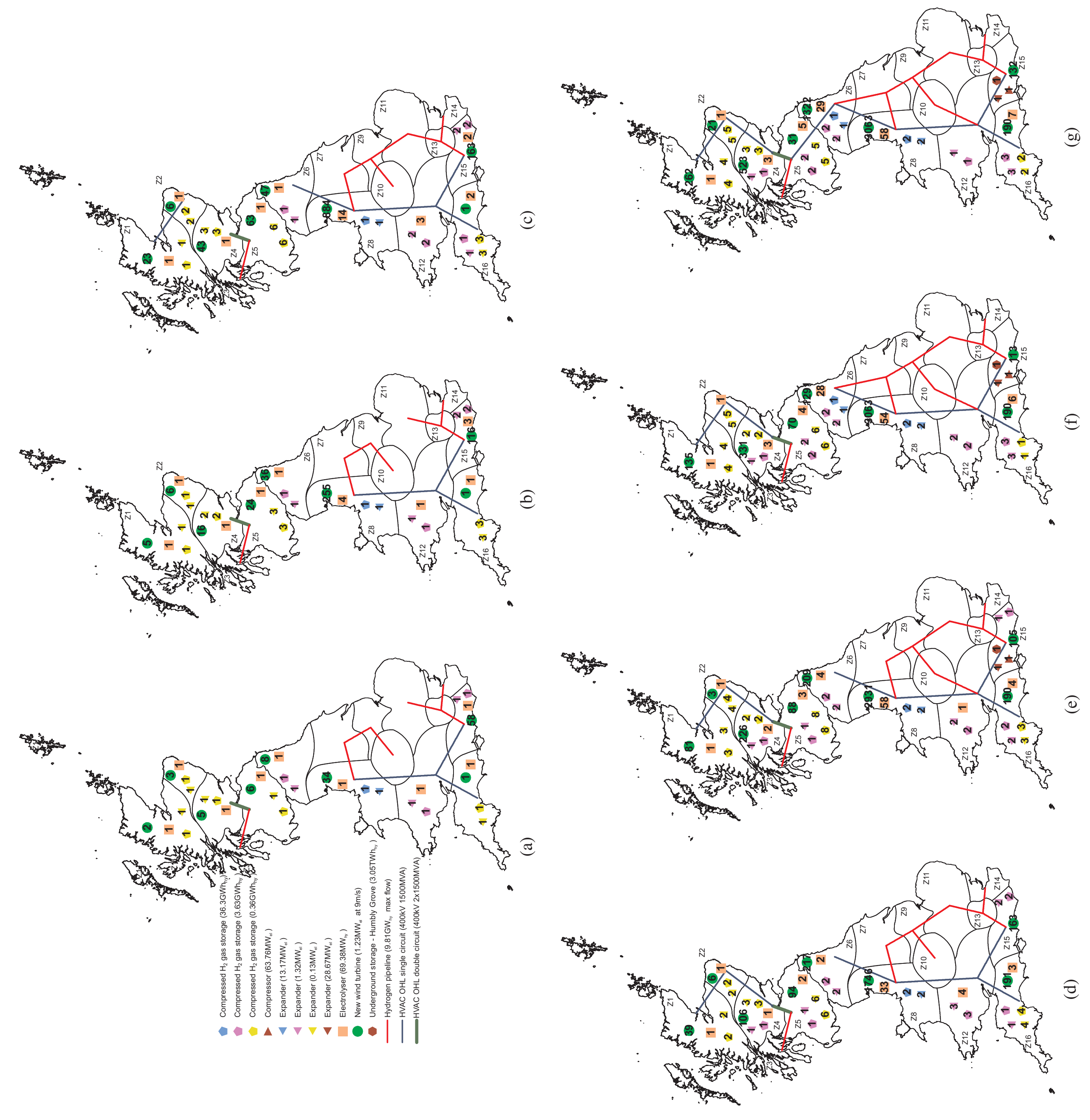


Fig. 11. Network configuration for net present cost optimisation: (a) 2015–2019; (b) 2020–2024; (c) 2025–2029; (d) 2030–2034; (e) 2035–2039; (f) 2040–2044; (g) 2045–2050.

Fig. 16 shows the spatial distribution of domestic heat and electricity demands as well as typical profiles for each. The spatial demand distributions were based on the data produced by Loughborough University [33] and the hourly distributions were based on the work of Sansom [50]. The heat demands (Fig. 16(c)) exhibit two distinct peaks, at similar times to the transport demands (see Fig. 6). Unlike the transport demands, however, the trough between the peaks is much lower and the weekday demands are very similar to the weekend demands. The demands are roughly 15 times larger than the transport demands. The electricity demands (Fig. 16(d)), on the other hand, are relatively flat from 07:00 h to 21:00 h with a slight peak in the evening. Again, the general shape of the profiles is the same for weekdays and weekends. The demands peak at about 2000 MW, nearly double that of the transport demands. Unlike the transport and heat demands, which fall to nearly zero at off-peak times, the electricity demands never fall below around 700 MW. The data in Fig. 16 are for 2015; future demands were obtained by assuming a fixed growth rate of 1% every year.

Figs. 17–19 present the results for the case where the net present value is maximised, subject to satisfying all of the domestic energy demands described above and the maximum allowed land area for wind turbines and biomass production. In this run, the prices of the energy service demands (V_{riy}) were taken to be £67.55/MWh for electricity, £37.5/MWh for heat and £33.9/MWh for hydrogen based on the data from Ref. [4]. To avoid showing too many symbols on the map, the

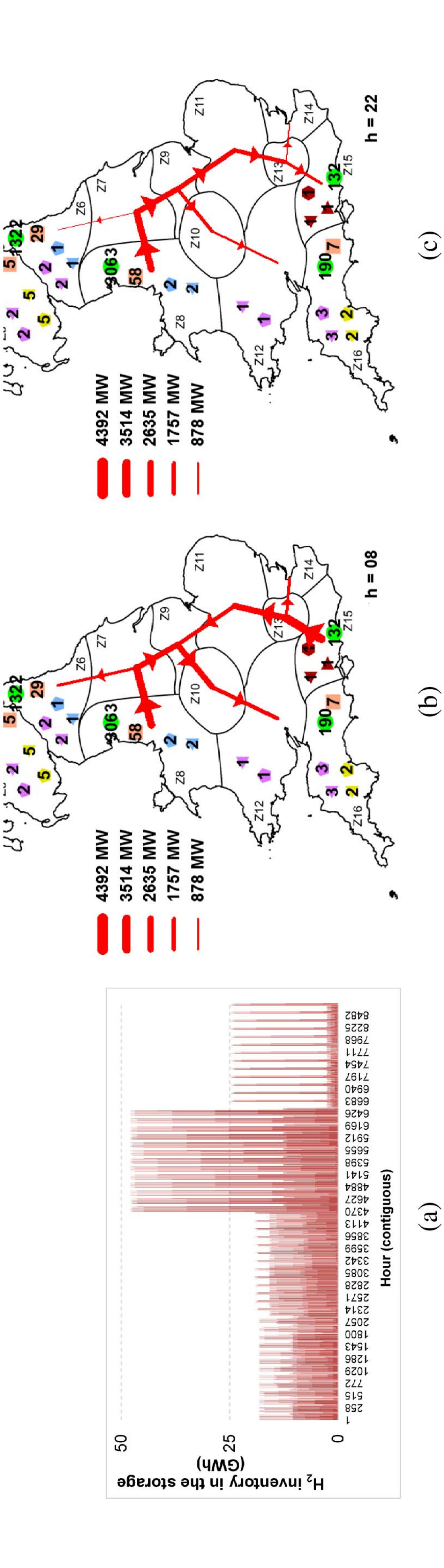


Fig. 12. Snapshot of some of the ways the system balances the demand for and supply of hydrogen in planning period 2045–2050: (a) large-scale storage of hydrogen in the Humbly Grove underground storage (the graph is showing the hourly inventory of hydrogen in the facility); and transmission of hydrogen via the pipeline network: (b) hydrogen is withdrawn from the Humbly Grove underground storage and transmitted to the neighbouring zones when the demands are high (e.g. at 8:00 h) and (c) hydrogen is transmitted back to zone 15 in order to replenish the underground storage when the demands are low (e.g. at 22:00 h). The symbols on the maps are defined in the legend of Fig. 11(a).

investments in technologies and infrastructures are shown first in Fig. 17, followed by the retirements in Fig. 18, and the final network for 2050 is presented in Fig. 19. In these figures, the coloured symbols represent conversion and storage technologies at different sizes (pentagons for compressed gas storage, sideways triangles for hydrogen compressors and expanders, upright triangles for electric heaters and domestic hydrogen-fired boilers, squares for electrolysers, diamonds for gas-fired CCGTs, dark red circles for SMR and green circles for wind turbines) with the number of installations shown on top of the symbols. The location of the symbols indicates the zone in which the technologies are installed. The coloured lines represent transmission connections between zones: red for hydrogen pipelines, brown for natural gas pipelines, blue for HVAC single-circuit 400 kV overhead lines and green for HVAC twin-circuit 400 kV overhead lines. In Fig. 17, the symbols and lines represent investments at the beginning of each planning period (each of the 7 periods represented by a subfigure); in Fig. 18, they represent retirements at the beginning of each period; and Fig. 19 shows the technologies present at the beginning of the final period.

In the superstructure considered in this paper (Fig. 5), mobility demands are satisfied through hydrogen. The results in Fig. 17 indicate that this requires investments in electrolysers, compressed hydrogen storage tanks and hydrogen pipelines, which can also be utilised effectively for generating, storing and transporting hydrogen in order to meet other energy service demands, in this case heat demands. The overall strategy is to use wind turbines in northern England and Scotland and use a combination of domestic electric heaters and domestic hydrogen boilers to satisfy the heat demands. It can be seen that the hydrogen boilers are initially invested in the south of England and further investments are made in future periods gradually moving north and eventually covering most of Scotland as well as all of England and Wales. Hydrogen is partly produced by electrolysers powered by electricity generated by the wind turbines and partly by an SMR plant in zone 12 from 2020 to 2045 and then in zone 6 from 2045 to 2050. Electricity demands in later periods are satisfied by three new CCGTs in zones 11, 15 and 16. In the first period, the transmission network is reinforced by installing new HVAC OHL electricity lines, both single and double circuits. Some new hydrogen pipelines are also built; these are augmented by further investments in 2020 and 2030, at which point all zones are connected to the hydrogen network apart from zones 1 and 2. There are no new investments in natural gas pipelines.

The retirements shown in Fig. 18 include existing technologies as well as the new investments shown in Fig. 17. The retirements of existing wind turbines can be seen in periods 2, 3 and 4, which are not replaced in many of the zones, the energy system focussing on wind turbines only in the southern half of Scotland and northern England; the retirements of the new turbines can be seen in the last three periods. The retirements of existing CCGTs can also be seen, as well as the SMR technology in zone 12.

The final network in 2050 is shown in Fig. 19. The full transmission network, including the existing electricity and natural gas networks, is shown. All zones are connected to the hydrogen network apart from zones 1 and 2, where hydrogen is produced locally by electrolysis to satisfy mobility demands. Wind turbines are present in the northern half of Great Britain and can be seen predominantly in southern Scotland, northern England, north Wales and north-east England; smaller numbers of wind turbines are installed throughout the remainder of Scotland. Hydrogen is produced by a large SMR plant in northern England and 2 large electrolysers in northern Scotland. Heat demands are satisfied through hydrogen boilers throughout GB apart from in zones 1 and 3, where only electric heaters are used. Electricity demands are met by the wind turbines in the northern half of GB and the 3 large CCGT plants, powered by natural gas, in the southern and eastern part of England.

Over the 35 years: the land area required for new wind turbines increases from 133 kha in 2015 to 155 kha in 2050, compared to the maximum suitable land area for wind turbines in each planning period

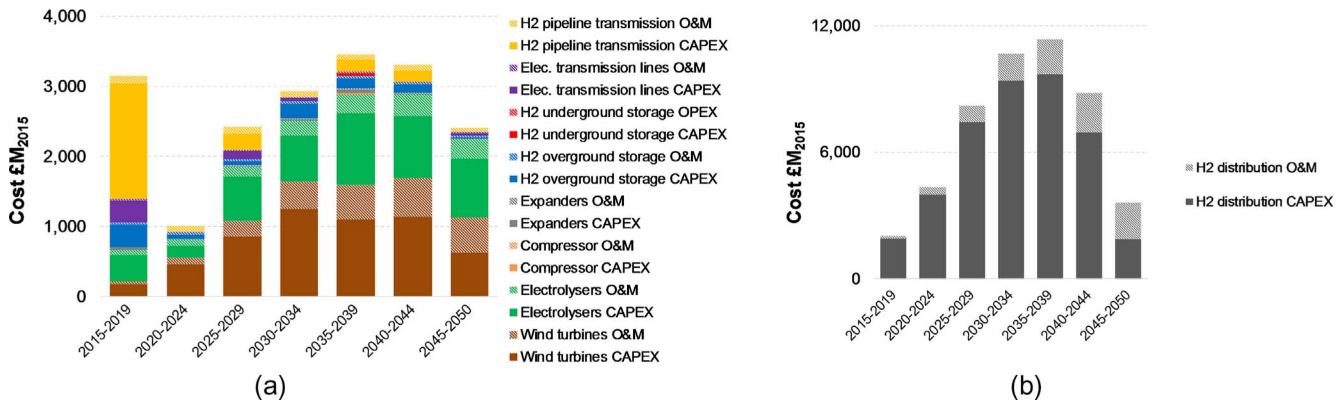


Fig. 13. Breakdown of costs for the least-cost scenario of meeting all domestic mobility demands using only wind power by 2050: (a) conversion, storage and transmission technologies; and (b) distribution technologies. The net present cost of the network is £67.7 billion and the selling price of the renewable hydrogen will need to be £68/MWh (£2.27/kg) in order to break even.

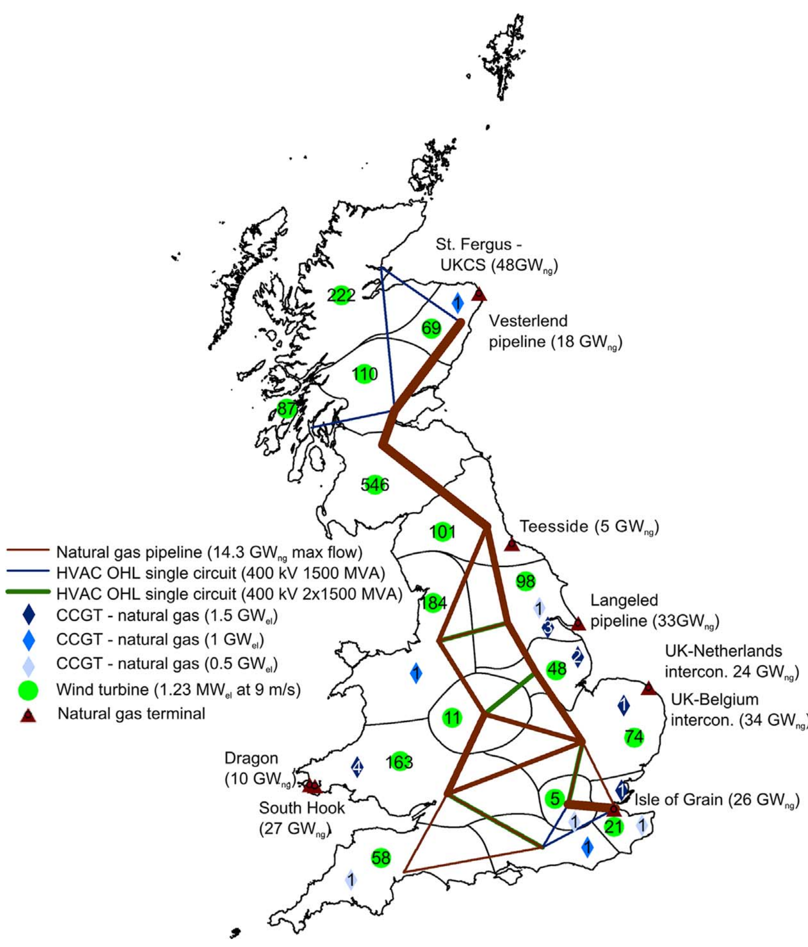


Fig. 14. Existing assets and natural gas availability.

of 425 kha; there is no production of biomass; 9133 TWh of natural gas is imported and no local production is required; and 3536 TWh of electricity is generated by wind turbines. These are utilised to satisfy 10083 TWh of demands broken down as 5687 TWh heat, 3400 TWh electricity and 996 TWh mobility (n.b. full penetration of fuel cell vehicles occurs in 2050, following the trajectory given by Fig. 10). The total NPV is £72 billion with CO_2 emissions of 2.07 Gt.

The final network in 2050 for the same problem but for minimising CO_2 emissions is shown in Fig. 20. The transmission network is larger than the max-NPV case, with more technologies utilised and more wind turbines and electrolysers installed. The hydrogen network is more extensive, in particular with more connections in the south of England; the electricity network has a higher capacity; and there is now a syngas pipeline network running from zone 4 down to zone 14, via some of the central zones in England and South Wales. Large electrolysers in Scotland and northern England and an SMR plant in zone 9 supply the hydrogen network. Domestic hydrogen boilers are the preferred technology for satisfying heat demands in all zones apart from the four southernmost zones in England, where a combination of hydrogen, syngas and natural gas CHPs are used to provide heat and electricity; a commercial syngas boiler is also utilised in zone 13. In these zones, a combination of domestic hydrogen boilers and district heating is used to satisfy the heat demands. Electric heaters are no longer used. All existing natural gas CCGT plants that retire are not replaced, thus this

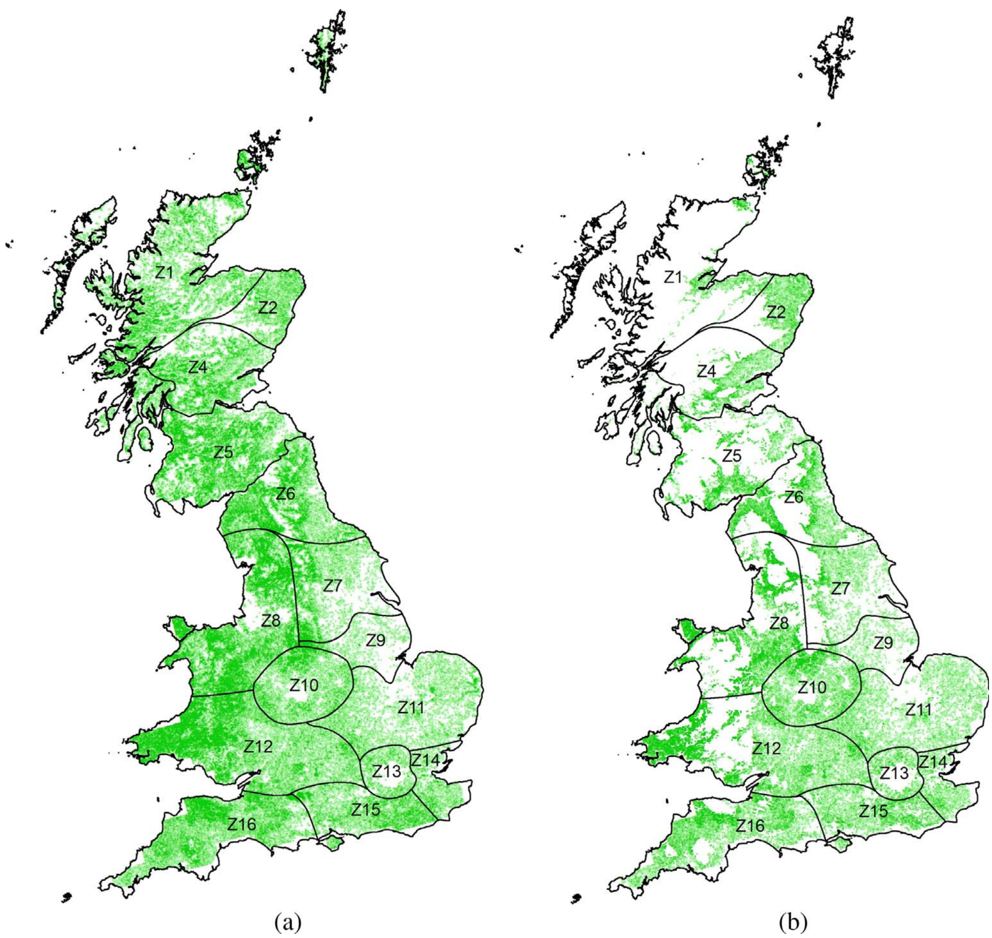


Fig. 15. Screening of grassland area suitable for cultivation of Miscanthus: (a) all grassland areas; (b) after elimination of areas with elevation > 250 m, slope > 15%, urban areas, roads, rivers, parks, protected areas, SSSI and areas of outstanding beauty.

technology is not present in the final network, where electricity is generated only by wind turbines and CHPs. The installation of wind turbines is now more uniformly spread throughout Great Britain, over 35 years generating about 1000 TWh more electricity than before. Two gasification plants are installed: one in zone 4 and the other in zone 12. These convert the modest amount of biomass grown into syngas, which is piped to the southern zones of England. Because of the limited land area allowed for wind turbines and biomass production, natural gas is still required in order to satisfy all of the domestic energy demands. The natural gas is used in SMR and CHPs but not in CCGT plants.

The overall results for this case are as follows. All of the land area allowed for wind turbines and biomass production is allocated: in particular, 425 kha of land area is occupied by new wind turbines and 313 kha is utilised for the production of biomass. Over the 35 years, 4564 TWh of electricity is generated by the wind turbines and 154 TWh of Miscanthus is produced. As before, no local natural gas is produced and this time 7569 TWh is imported. Similar to the previous case, 10083 TWh of domestic energy demands were required to be satisfied. The CO_2 minimisation case resulted in an NPV of −£17bn and CO_2 emissions of 1.72 Gt. The achievable reduction in CO_2 emissions is limited by the maximum land area allowed for wind turbines installation and biomass production.

The final case, shown in Fig. 21, has the objective of maximising energy production subject to not using any natural gas. Unlike the two previous cases, in which all domestic demands for electricity, heat and mobility have to bet met, in this case the optimisation can determine how much of each demand to satisfy. The results give an indication of the best form of energy to produce given the available renewable primary sources. As expected, the allowed land areas for wind turbines and biomass are allocated fully. However, without natural gas being available, only 4598 TWh of demands can be met: 2116 TWh for electricity (62% of the total domestic electricity demands) and 2481 TWh heat (44% of the total domestic heat demands). As no hydrogen technologies and infrastructure are installed, none of the mobility demands are satisfied. Heat demands are met by electric heaters this time (since the lower overall efficiency of producing hydrogen would result in a lower total production of energy) apart from in two zones where there are syngas CHPs and district heating networks installed. The syngas is produced in zone 16 and transported via syngas pipelines. Other than that, the transmission network is entirely electricity (Fig. 21 includes the existing electricity network presented in Fig. 14). The NPV of this energy system is £24 billion and the CO_2 emissions are 4.6 Mt (recall that not all domestic energy demands are satisfied in this case).

On the whole, different objectives resulted in very different energy networks. However, in all four cases, significant new wind turbine capacity is installed and expansion of the existing electricity networks is required; new investment in the natural gas network is not needed. All of the cases feature multiple energy vectors whose networks are closely integrated. All zones are interconnected via different transmission networks and can therefore share resources and facilities. The resulting networks are mixtures of centralised and distributed generation: they include large scale technologies such as CCGT and SMR plants that can supply many zones in Great Britain but there are also many local production technologies, such as wind turbines and electrolysers, distributed across the country. These smaller-scale technologies are also connected to the transmission networks and can meet demands in other zones if necessary. The transmission and storage technologies give more flexibility to the system and can mitigate the intermittency of renewables by enabling resources to be utilised when and where they are needed.

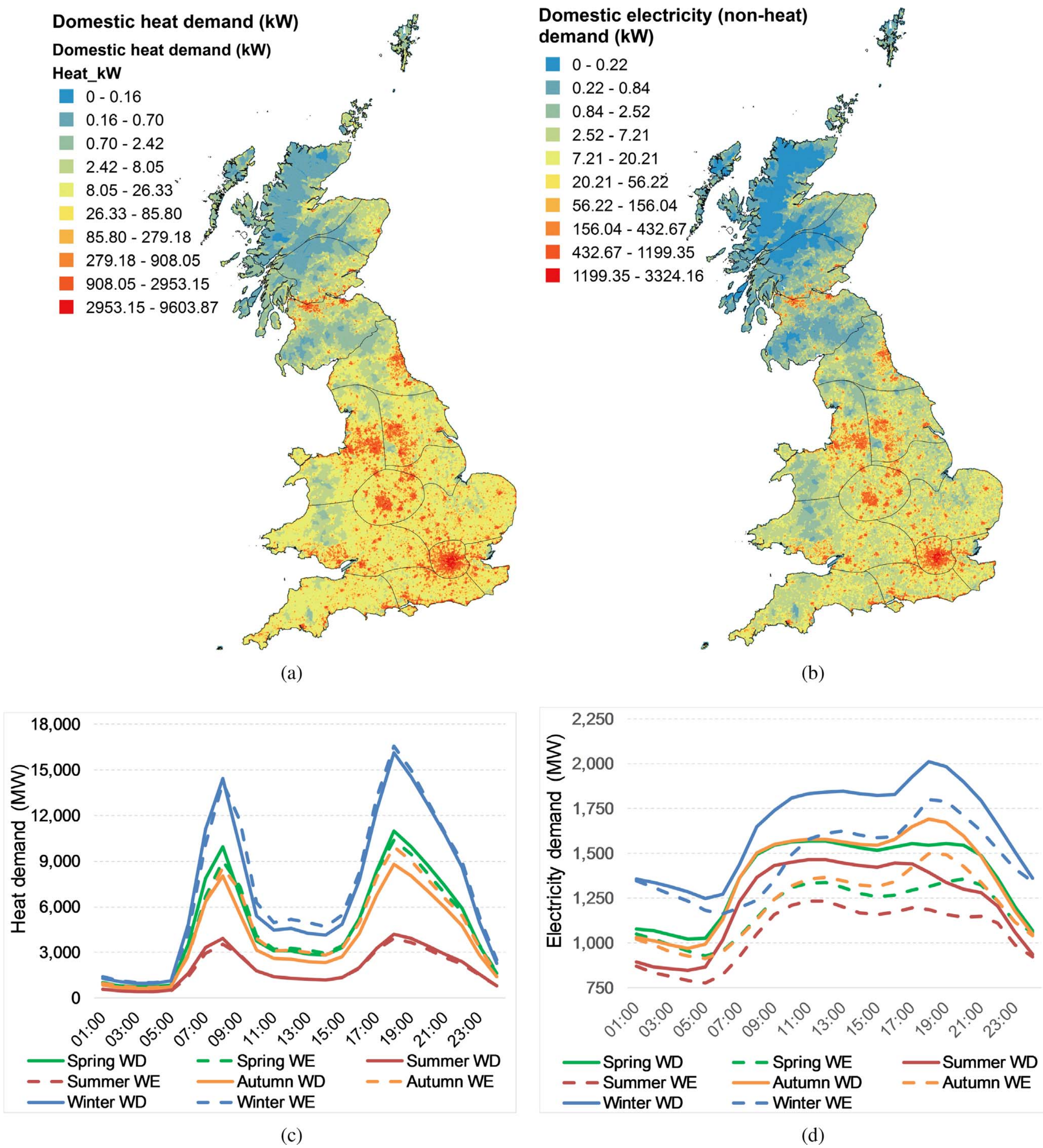


Fig. 16. Representative data for domestic heat and electricity demands: spatial distribution of (a) heat and (b) electricity demands [33]; and temporal variability of (c) heat and (d) electricity demands for zone 13 (similar time-series profiles were derived for the other zones) [50].

Finally, if there are significant compulsory demands for hydrogen, such as in the NPV maximisation and CO_2 emissions minimisation cases where the domestic mobility demands are to be met by hydrogen-powered fuel cell vehicles, then hydrogen is the preferred energy vector over natural gas for satisfying heat demands. If natural gas cannot be used and energy can be generated only from wind power and biomass, electricity and syngas are the preferred energy carriers for satisfying electricity and heat demands. The available land area for wind turbines and biomass is the limiting constraint on the CO_2 emissions reduction that can be achieved.

7. Conclusions

This paper presented a multi-objective MILP model for integrated multi-vector energy networks and applied it to a number of scenarios for Great Britain. The model has a flexible representation of the inherent spatial nature of the problem whereby the study region is divided into a number of zones within which different properties are defined: time-dependent profiles for primary resource availabilities and energy service demands, available area for the location of various facilities (e.g. wind turbines, cultivation of biomass, natural gas terminals, etc.). Time is represented on multiple scales: hourly, daily and

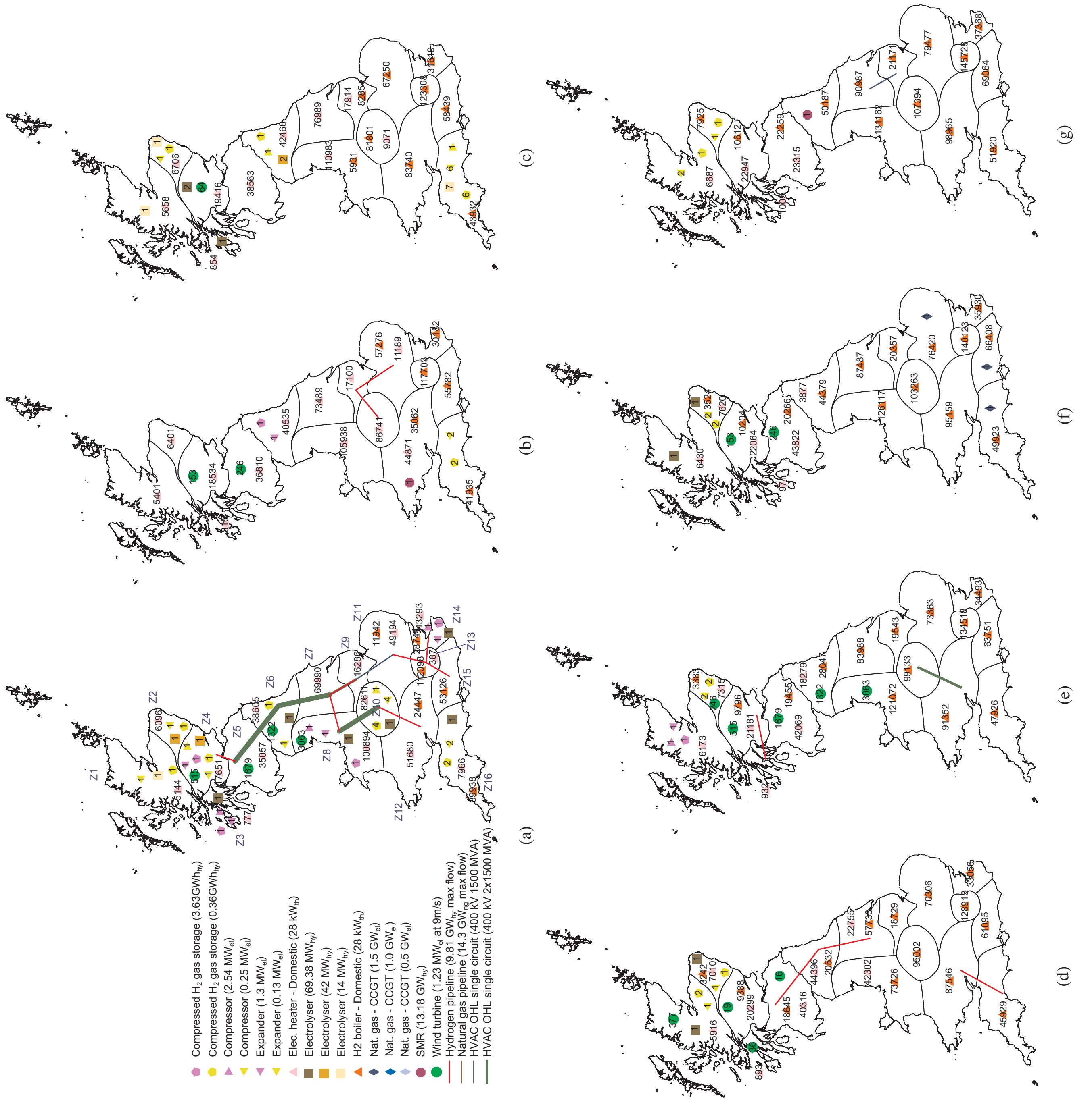


Fig. 17. Staged investment in technologies and infrastructures for the case of maximising the net present value: (a) 2015–2019; (b) 2020–2024; (c) 2025–2029; (d) 2030–2034; (e) 2035–2039; (f) 2040–2044; (g) 2045–2050.

seasonal levels for operational issues and long-term planning periods for investment decisions and technology retirements. The model can optimise both strategic decisions of investments over many decades and tactical operational decisions at the hourly level, such as deciding how much energy to store, when and where, in response to fluctuations in renewable generation. The model also has a detailed representation of storage and transport of the energy carriers, accounting for costs, emissions, energy requirements and losses/efficiencies, and includes an inventory balance that tracks the level of stored resources at the hourly level over the entire time horizon. Unlike many models in the literature, which consider one or two different energy vectors, this is a true multi-vector model: diverse vectors with very different properties can be considered, such as wind generation with hourly dynamics compared with the much longer dynamics of biomass, for which harvesting takes place on a seasonal or yearly time scale. The model is data-driven and allows any technology, energy vector or network to be added simply by specifying their properties in the database: the data define the superstructure of pathways and interactions between the energy vectors, and the model selects those that will optimise the whole network.

The model was used to consider a number of scenarios for Great Britain, based on satisfying domestic demands for heat, electricity and mobility using three different primary resources: natural gas, biomass and on-shore wind power. Potential vectors included syngas, electricity, hydrogen and natural gas. A wide variety of technologies were considered, including commercial and domestic boilers (fired by syngas, hydrogen and natural gas), electric heaters, district heating networks, CHP, CCGT plants

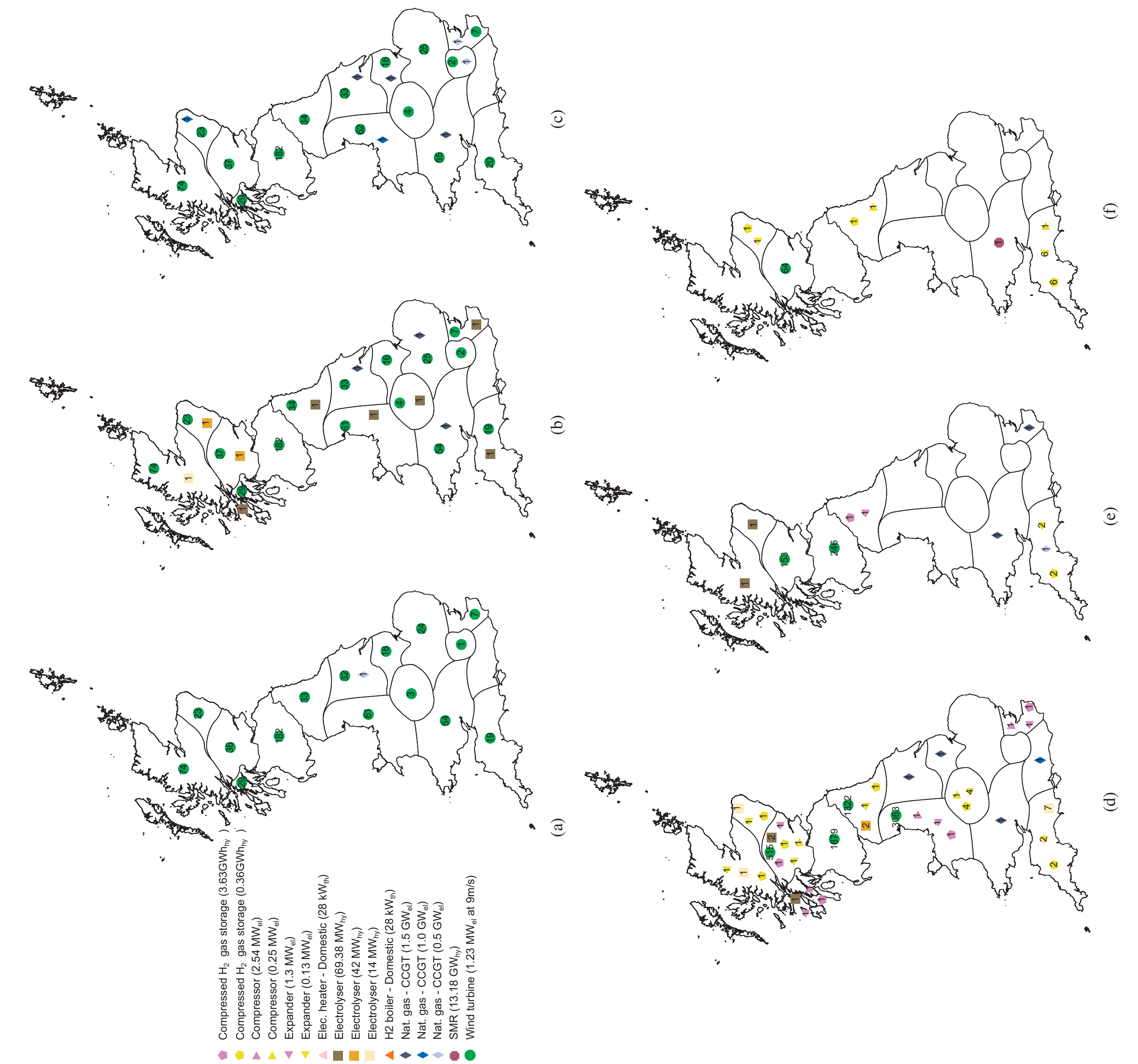


Fig. 18. Retirement of technologies and infrastructures for the case of maximising the net present value: (a) 2020–2024; (b) 2025–2029; (c) 2030–2034; (d) 2035–2039; (e) 2040–2044; (f) 2045–2050.

(natural gas and syngas fired), SMR to generate hydrogen from natural gas, biomass gasification, fuel cells, electrolysers and explicit consideration of the expanders and compressors required for compressed-gas storage. Natural gas, syngas and hydrogen can be stored in tanks of three different sizes as well as in underground salt caverns and depleted oil and gas reservoirs. Pipelines can be installed to transport hydrogen and syngas between regions, and the natural gas grid can also be reinforced by installing further pipeline capacity. Similarly, the existing electricity network can be extended by installing a variety of transmission lines. GIS analysis was performed to identify the area in each zone suitable for siting wind turbines and planting biomass, based on a number of technical and socio-environmental constraints, which was used to constrain their deployment in all case studies.

The case studies were designed to examine the different networks (both in terms of their evolution from 2016 to 2050 and final network) resulting from different objectives. The first case determined the design and operation of integrated renewable hydrogen and electricity networks in order to meet domestic mobility demand, in the form of hydrogen for fuel cell vehicles. The second and third cases optimised the design and operation of integrated electricity-natural gas-hydrogen-syngas networks to satisfy all of the domestic demands for mobility, heat and electricity using wind power, biomass and natural gas as possible primary energy resources. Case 2 maximised the net present value over the 35 year time horizon whereas case 3 minimised CO_2 emissions. Finally, case 4 determined the maximum amount of energy that can be generated from wind and biomass only (i.e. no utilisation of natural gas) and which demands (heat, electricity or hydrogen for mobility) to satisfy in order to do so.

Overall, the networks are very different for different objectives, but in all cases networks for different energy vectors are present and integrated. This suggests that integrated multi-vector energy networks could play an important role in future energy systems. Another feature common to all cases is that in 2050 all zones are connected and there is a mixture of different technologies at different scales: large-scale

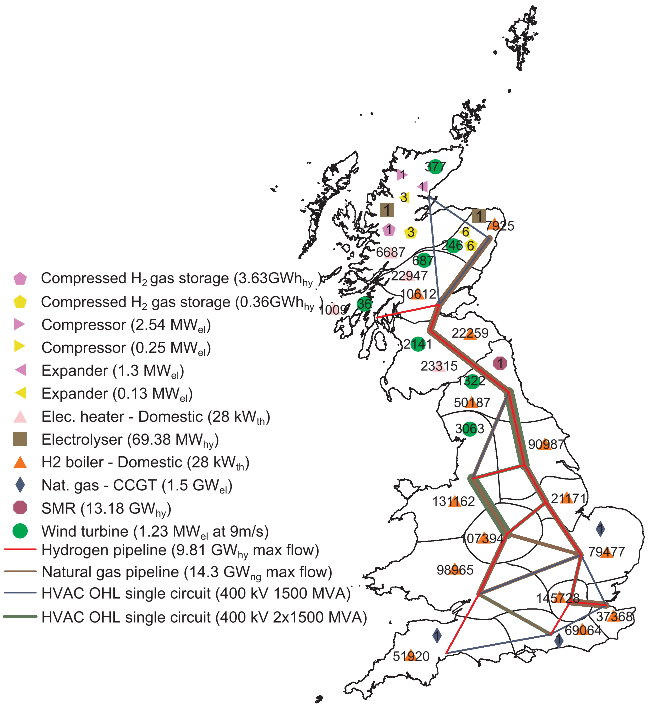


Fig. 19. Optimal structure of the network in 2050 for the net present value maximisation scenario.

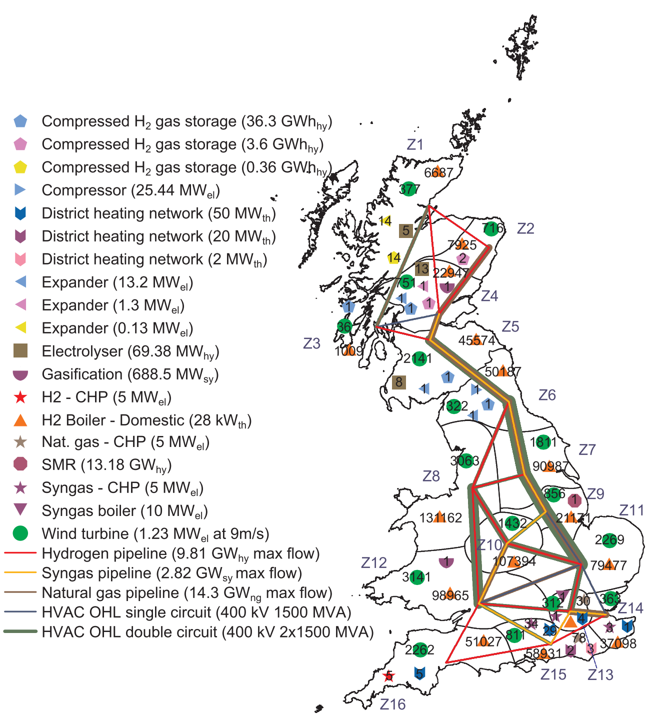


Fig. 20. Optimal structure of the network in 2050 for the CO_2 emissions minimisation scenario.

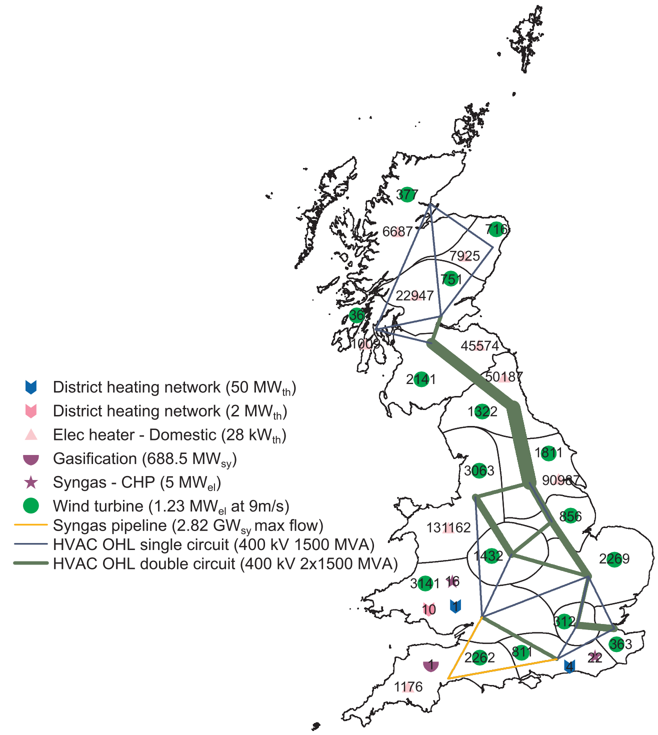


Fig. 21. Optimal structure of the network in 2050 for the scenario of maximising energy production subject to not using any natural gas.

technologies, such as CCGT and SMR plants, provide centralised generation while smaller-scale technologies, such as wind turbines and electrolysers distributed across the country, are all connected to the energy networks, and thus can cover shortfall in supply in other zones whenever they have excess generation. This indicates that no single strategy is likely to be the solution for the future energy system: hybrid strategies of centralised and distributed systems comprising technologies of diverse types and scales, strategically located across the country, will be required. A mixture of primary resources will also be needed and the relative utilisation of each depends on the objective. If the objective is economic, not all of the suitable areas for wind turbines and biomass may be utilised, natural gas being a cheaper option. Minimising emissions favours the utilisation of wind power and biomass, as expected, but the degree of decarbonisation that can be achieved is limited by the available suitable land areas; some natural gas may still be required to satisfy all domestic energy demands, with SMR and CHP being the preferred technologies over CCGT. If natural gas is prohibited and only renewables, such as wind power and biomass, are used to maximise renewable energy production then they are best utilised to produce electricity and syngas to satisfy electricity and heat demands. If transport demand is satisfied through hydrogen, then it is also advantageous to satisfy heat demands using hydrogen.

Future areas of study include consideration of other alternative technologies for satisfying mobility demands, such as vehicles powered by electricity (e.g. see [51] for the authors' first step in this direction), biofuels, natural gas, as well as addition of various technologies for CO_2 mitigation, such as CCS and CCU. Extending the network superstructure is simply a matter of adding these new technologies and resources (and the required data for their properties) to the database, with little or no changes required in the mathematical formulation. Other areas where the model can be extended is hydrogen injection into the natural gas grid, pipeline storage (linepack) and offshore wind turbines (e.g. see [52] for the author's preliminary work in this area). These are the next major steps in the development of the model. In addition, uncertainties have not been taken into account, the main objective being first to develop a suitable deterministic model. Although others have considered uncertainty, for example in hydrogen supply chain models, the models are considerably simpler than this one, containing no hourly dynamics and often not considering wind power, and therefore the uncertainty has only been applied to the hydrogen demands. Considering uncertainty in the wind profiles is a considerably more challenging problem that will need to be addressed. In the case studies presented in this paper, very conservative assumptions were used in order to mitigate some of the uncertainties (e.g. selecting representative wind profiles that exhibit the most variability). This work demonstrates the important first steps of developing a deterministic model with sufficient detail to examine the intricate role that energy storage and load management will play in future energy systems where intermittent renewables satisfy a significant proportion of the energy demands. Models with annual averages are not suitable for this and those models that do have a fine temporal resolution have not considered a long time horizon of many decades or considered uncertainties in wind speed profiles.

Finally, the presented MILP model has a wide range of applications, not only to integrated energy systems but to other supply chains involving conversion, storage and transport of resources, and in particular ones that include circular and interlinking chains common to circular economy problems.

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Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.apenergy.2017.09.055.

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